4-1 Notes

Math III
~~Notes 5-2~~ Exponent/Logarithm Review

5-1 Notes

Name * Key *
Date________ Period ______

Exploring Exponential Models

An exponential Function is a function with the general form $y=ab^x$,
where x is a real number, $a \neq 0$, $b > 0$, and $b \neq 1$.

You can use an exponential function with $b>1$ to model growth.

When $b>1$, b is the growth factor.

You can use an exponential function with $0<b<1$ to model decay.

When $0<b<1$, b is the decay factor.

Writing an Exponential Function

Write an exponential function $y = ab^x$ for a graph that includes (2, 2) and (3, 4).

$y=ab^x$	**Use the general form.**
$2=ab^2$	**Substitute for x and y using (2, 2)**
$2/b^2=a$	**Solve for a.**
$y=ab^x$	**Use the general form.**
$4=\frac{2}{b^2}b^3$	**Substitute for x and y using (3, 4) and for a using $\frac{2}{b^2}$.**
$4=2b^{3-2}$	**Division Property of Exponents**
$4=2b^1$	**Simplify.**
$2=b$	**Solve for b.**
$a=2/b^2$	**Use you equation for a.**
$a=2/2^2$	**Substitute 2 for b.**
$a=2/4=1/2$	**Simplify.**
$y=1/2\cdot 2^x$	**Substitute $\frac{1}{2}$ for a and 2 for b in $y = ab^x$.**

$4=\frac{2b^3}{b^2}$

* This can also be done in your calculator.

→ Stat, Edit, L_1, L_2, Stat, Calc, Exp. Regression

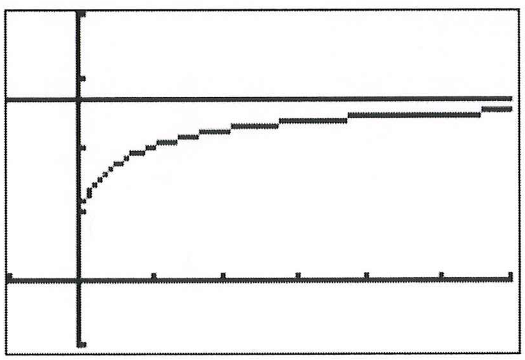
Properties of Exponential Functions

Families of Exponential Functions	
Parent function:	$y = ab^x$
Stretch $(\lvert a \rvert > 1)$ Shrink $(0 < \lvert a \rvert < 1)$ Reflection $(a < 0)$ over the x-axis	$y = ab^x$
Translation (horizontal by h, vertical by k)	$y = b^{x-h} + k$
Combined	$y = a \cdot b^{x-h} + k$

The Number e

The following is the graph of $y = \left(1 + \frac{1}{x}\right)^x$. One of the graph's asymptotes is $y = e$, where e is an irrational number approximately equal to 2.71828... .

$y = e \rightarrow$



X

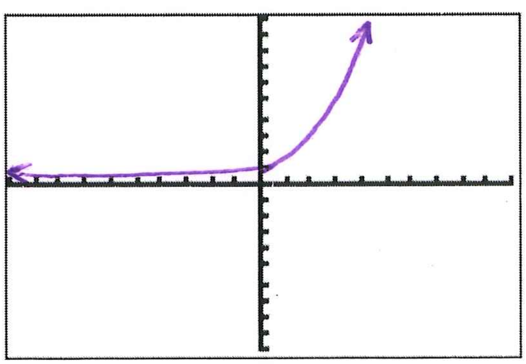
y-axis

* Remember - what is an asymptote?

Evaluating e^x

Graph $y = e^x$. Evaluate e^2 to four decimal places.

$e^2 = 7.3891$



Continuous Compounded Interest Formula

(A) ~~Starting~~ Amount after t years

(r) rate of interest

(t) time

$$A = Pe^{rt}$$

(P) Principal (Initial Amount)

* e is a number

Example

Suppose you invest \$1050 at an annual interest rate of 5.5% compounded continuously. How much money, to the nearest dollar, will you have in the account after five years?

$A = Pe^{rt}$

$A = 1050\left((e)^{(.055)(5)}\right)$

$= \$1382.36$

Logarithm * What is a logarithm?

The logarithm to the base *b* of a positive number *y* is defined as follows:

If $y = b^x$ then $\log_b y = x$

Writing in Logarithmic Form

Write $25 = 5^2$ in logarithmic form.

$5^2 = 25$

$\log_5 25 = 2$

$3^5 = 243$

$\cancel{\log_3 5 = 243}$

$\log_3 243 = 5$

Evaluating Logarithms

Evaluate $\log_8 16$

$\log_8 16 = x$

2 ways to solve

① $\frac{(\log 16)}{(\log 8)} = 1.3333$

② math, Log Base
fill in missing
values

~~52~~

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Notes ~~5-3~~ Properties of Logarithms　　　　　　Date________ Period ______

Properties of Logarithms

For any positive numbers, M, N, and b, $b \neq 1$,

$\log_b MN = \log_b M + \log_b N$	Product Property
$\log_b \frac{M}{N} = \log_b M - \log_b N$	Quotient Property
$\log_b M^x = x\log_b M$	Power Property

Change of Base Formula

For any positive numbers, M, b, and c, with $b \neq 1$ and $c \neq 1$,

$$\log_b M = \frac{\log_c M}{\log_c b}$$

Why is the Change of Base Formula helpful?

Use the Change of Base Formula to evaluate $\log_3 15$.

$\frac{\log_{10} 15}{\log_{10} 3}$	**Use the Change of Base Formula**
$= 2.46497$	**!!! Use a calculator !!!**

Math III 4 - 3 Notes

~~5-3~~ Notes ~~Notes 5-4~~ Exponential and Logarithmic Equations

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Exponential and Logarithmic Equations

An equation of the form $b^{cx} = a$, where the exponent includes a variable, is an exponential equation. If *m* and *n* are positive and *m* = *n*, then $\log m = \log n$.

You can therefore solve an exponential equation by taking the logarithm of each side of the equation.

Example 1: Solving an Exponential Equation

Solve $7^{3x} = 20$

$$\log 7^{3x} = \log 20$$

$$\frac{3x \log 7}{3 \log 7} = \frac{\log 20}{3 \log 7}$$

$$x = .5132$$

You Try:

Solve each equation. Round to the nearest ten-thousandth. Check your answers.

a. $3^x = 4$

$$\frac{x \log 3}{\log 3} = \frac{\log 4}{\log 3}$$

$$x = 1.2619$$

b. $6^{2x} = 21$

$$\frac{2x \log 6}{2 \log 6} = \frac{\log 21}{2 \log 6}$$

$$x = .8496$$

c. $3^{x+4} = 101$

$$(x+4)\log 3 = \log 101$$

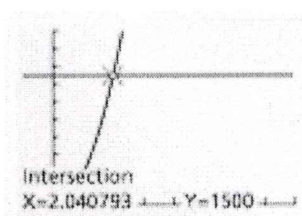
$$x \log 3 + 4 \log 3 = \log 101$$

$$\frac{x \log 3}{\log 3} = \frac{\log 101 - 4 \log 3}{\log 3}$$

$$x = .2009$$

Example 2: Solving an Exponential Equation by Graphing

Solve $6^{2x} = 1500$



You Try:

$$y_1 = 6^{2x}$$

$$y_2 = 1500$$

2nd, trace, intersection, enter 3 times

Solve $11^{6x} = 786$ by graphing.

$x = .4634$

Example 3: Real-World Connection

<u>Zoology</u> Refer to the photo. Write an exponential equation to model the decline in the population. If the decay rate remains constant, in what year might only five peninsular bighorn sheep remain in the United States?

Real-World Connection

The U.S. population of peninsular bighorn sheep was 1170 in 1971. By 1999, only 335 remained.

	L1	L2
0	1971	1170
28	1999	335

Stat, Calc, Exp.Reg

$y = 1170(.9563)^x$

You Try:

The population of peninsular bighorn sheep in Mexico was approximately 6200 in 1971. By 1999, about 2300 remained. Determine the year by which only 200 peninsular bighorn sheep might remain in Mexico.

0 1971 → 6200

28 1999 → 2300

$\boxed{2068}$

$y = 6200(.9652)^x$

$\underset{y_1}{200} = \underbrace{6200(.9652)^x}_{y_2}$

To evaluate a logarithm with any base, you can use the **Change of Base Formula.**

Property **Change of Base Formula**

For any positive numbers, M, b, and c, with $b \neq 1$ and $c \neq 1$,

$$\log_b M = \frac{\log_c M}{\log_c b}$$

Example 4: U

Ex4) Use the Change of Base Formula to evaluate $\log_3 15$. Then convert $\log_3 15$ to a logarithm in base 2.

$\log_3 15 = 2.4650$

$2.4650 = \log_2 x$ (circle rule)

$2^{2.4650} = x$ $x = 5.5212$

An equation that includes a logarithmic expression, such as $\log_3 15 = \log_2 x$ in Example 5, is called a Logarithmic Equation.

You Try:

Evaluate $\log_5 400$ and convert it to a logarithm in base 8.

$\log_5 400 = 3.7227$

$3.7227 = \log_8 x$

$8^{3.7227} = x$

$2301.1 = x$

Example 5: Solving a Logarithmic Equation

Solve $\log(3x + 1) = 5$

$10^5 = 3x+1$

$100000 = 3x+1$

$\frac{99999}{3} = \frac{3x}{3}$ $x = 33333$

You Try:
Solve $\log(7 - 2x) = -1$. Check your answer.

$10^{-1} = 7-2x \Rightarrow 0.1 = 7-2x \Rightarrow -6.9 = -2x$

$x = 3.45$

Example 6: Using Logarithmic Properties to Solve an Equation
Solve $2\log x - \log 3 = 2$

$\log \frac{x^2}{3} = 2$

$10^2 = \frac{x^2}{3}$

$(3)100 = \frac{x^2}{3}(3)$

$300 = x^2$

$\pm 10\sqrt{3} = x$

You Try:

Solve log6 - log3x = -2

$$\log \frac{6}{3x} = -2$$

$$10^{-2} = \frac{6}{3x}$$

$$(3x)0.01 = \frac{6}{3x}(3x)$$

$$0.03x = 6$$

$$x = 200$$

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Math III
~~Notes 5-5~~ Natural Logarithms

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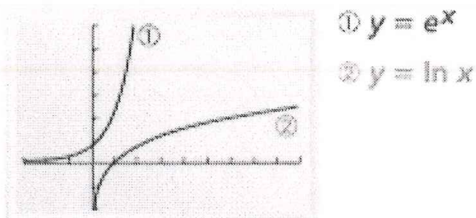
Natural Logarithms

Definition **Natural Logarithmic Function**

If $y = e^x$, then $\log_e y = x$, which is commonly written as $\ln y = x$.

The natural logarithmic function is the inverse, written as $y = \ln x$.



The properties of <u>common</u> logarithms apply to <u>natural</u> logarithms also.

* power, product, quotient

Example 1: Simplifying Natural Logarithms

Write 3 ln 6 - ln 8 as a single natural logarithm.

$\ln 6^3 - \ln 8$

$\ln \frac{6^3}{8} \Rightarrow \ln \frac{216}{8} \Rightarrow \ln 27$

You Try:

a. 5ln2 - ln4

$\ln \frac{2^5}{4}$

$\ln \frac{32}{4} \Rightarrow \ln 8$

b. $3\ln x + \ln y$

$\ln x^3 + \ln y \Rightarrow \ln(x^3 \cdot y)$

<u>Natural</u> <u>logarithms</u> are useful because they help express many relationships in the physical world.

Example 2: Real-World Connection

Space A spacecraft can attain a stable orbit 300 km above Earth if it reaches a velocity of 7.7 km/s. The formula for a rocket's maximum velocity v in kilometers per second is $v = -0.0098t + c \ln R$. The booster rocket fires for t seconds and the velocity of the exhaust is c km/s. The ratio of the mass of the rocket filled with fuel to its mass without fuel is R. Suppose a rocket used to propel a spacecraft has a mass ratio of 25, an exhaust velocity of 2.8 km/s, and a firing time of 100 s. Can the spacecraft attain a stable orbit 300 km above Earth?
Let $R = 25, c = 2.8,$ and $t = 100$. Find v.

$$V = -0.0098(100) + 2.8 \ln 25$$
$$= -0.98 + 2.8(3.219)$$
$$\approx 8.0$$

yes stable orbit

You Try:

A booster rocket for a spacecraft has a mass ratio of about 15, an exhaust velocity of 2.1 km/s, and a firing time of 30 s. Find the maximum velocity of the spacecraft. Can the spacecraft achieve a stable orbit 300 km above Earth?

$$V = -0.0098(30) + 2.1 \ln 15$$
$$\approx 5.4$$; no it can not

You can use the properties of logarithms to solve natural logarithms equations.

Example 3: Solving a Natural Logarithmic Equation

Solve $\ln(3x+5)^2 = 4$

$e^{\ln(3x+5)^2} = e^4$

$(3x+5)^2 = 54.60$

$3x+5 = \sqrt{54.60}$

$3x+5 = \pm 7.39$

$3x+5 = 7.39$
$-5 \quad -5$

$\frac{3x}{3} = \frac{2.39}{3}$

$x = .797$

$3x+5 = -7.39$
$-5 \quad -5$

$\frac{3x}{3} = \frac{-12.39}{3}$

$x = -4.13$

You Try:

a. $\ln x = 0.1$

$x = 1.105$

b. $\ln(3x-9)=21$

$x = 439{,}605{,}247.8$

c. $\ln\left(\frac{x+2}{3}\right) = 12$

$x = 488262.37$

You can use natural logarithms to solve exponential equations.

Example 4: Solving an Exponential Equation

Use Natural Logarithms to solve $7e^{2x} + 2.5 = 20$.

$7e^{2x} + 2.5 = 20$
$-2.5 \quad -2.5$

$\frac{7e^{2x}}{7} = \frac{17.5}{7} \rightarrow e^{2x} = 2.5 \rightarrow 2x = \ln 2.5 \rightarrow x = .4581$

You Try:

a. $e^{x+1} = 30$

$x = 2.401$

b. $e^{\frac{2x}{5}} + 7.2 = 9.1$

$e^{\frac{2x}{5}} + 7.2 = 9.1$

$x = 1.605$

Example 5: Real-World Connection

An investment of $100 is now valued at $149.18. The interest rate is 8%, compounded continuously. About how long has the money been invested?

$A = Pe^{rt}$

$\frac{149.18}{100} = \frac{100e^{.08t}}{100}$

$1.4918 = e^{.08t}$

$\frac{\ln 1.4918}{.08} = \frac{.08t}{.08}$

$t = 4.9999$

$t \approx 5$

You Try:

An initial investment of $200 is worth $315.24 after 7 years of continuous compounding. Find the interest rate.

$315.24 = 200e^{7r}$

$r = .065$

$r = 6.5\%$