2-1 Notes

Math III

Notes ~~3-1~~ Graphing Polynomial Functions

Name * KEY *

Date________________ Period ________

Each of these functions is a polynomial function:

a) $f(x) = x^3$ b) $f(x) = 7x^2 - 2x$ c) $f(x) = 5x^3 + 2x - 12$ d) $f(x) = \frac{3}{4}x^2 - 5x + 7.9$

Each of these functions is NOT a polynomial function:

e) $f(x) = 23x^{2/3}$ f) $f(x) = 26*(.5)^x$ g) $f(x) = \sqrt{x}$ h) $\frac{1}{x^3}$

A polynomial function
- has at least one term
- no variables in the exponents
- no fractional exponents
- no negative exponents

Definition of a Polynomial Function –
A polynomial function is a sum of power function
whose exponents are non-negative integers

Standard Form of a Polynomial Function
$g(x) = 3x^2 + 4x^5 + x - x^3 + 6$ is a polynomial function. (Recall that even 6 can be written as $6x^0$.) However, g(x) is not written in standard form. The standard (or general) form for g(x) is $g(x) = 4x^5 - x^3 + 3x^2 + x + 6$

For a polynomial to be in standard form, the exponents must be in descending order

Polynomials have the following characteristics: Example ($4x^5 - x^3 + 3x^2 + x + 6$)

(5th degree) a) Degree: the highest power of an exponent

(5) b) # of Terms: # of monomial terms

($4x^5$) c) Leading Term: the monomial term of the highest degree

(4) d) Leading Coefficient: the coefficient of the leading term

(6) e) Constant Term: the term without a variable

Ex 1) Are the following functions polynomial functions? If so, put them in standard form and state a) the degree, b) # of terms, c) leading term, d) leading coefficient, and e) constant term. If not, then tell why not.

a) $y = 3x^2 + 5$

a) 2nd degree
b) 2
c) $3x^2$
d) 3
e) 5

b) $y = 4x^2 - 7\sqrt{x^9} + 10$

$-7x^{9/2}$
Not a polynomial function b/c of fractional exponent

c) $y = 5^x - 2$

5^x
No, have a variable as an exponent

d) $y = 7t^2 - 8t + 6$

a) 2nd degree
b) 3 terms
c) $7t^2$
d) 7
e) 6

e) $y = 3.1 - 8x^2 + 5x^5 - 12.3x^4$

a) 5th
b) 4 terms
c) $5x^4$
d) 5
e) 3.1

f) $y = x^2 + 5x$

a) 2nd degree
b) 2 terms
c) x^2
d) 1
e) 0

Large-Scale Behavior of Polynomial Functions

Ex.2) Consider the polynomial $f(x) = 3x^2 + x + 6$.

a) What is the value of f(x) when x=2? $3(2)^2+2+6 = 20$
What is the value of just the leading term when x=2? 12
Notice that when x=2, the value of the leading term makes up $12/20$ OR 60% of the value of the whole polynomial.

b) What is the value of f(x) when x=100? $3(100)^2 + 100 + 6 = 30,106$
What is the value of just the leading term when x=100? 30,000
Notice that when x=100, the value of the leading term makes up $\frac{30,000}{30,106} = 99.6\%$ of the value of the whole polynomial.

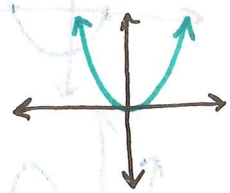
In general, if x is large enough, we can estimate the value of a polynomial by calculating the value of just the leading term.
Thus, for the function f(x), for large values of x, the algebraic expression "$3x^2$" is approximately equal to $3x^2 + x + 6$.

If this is what happens numerically, what would you think would happen graphically?

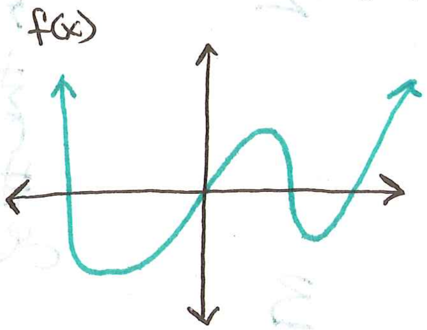
Ex 1: Compare the graphs of the polynomial functions f, g, and h given by...

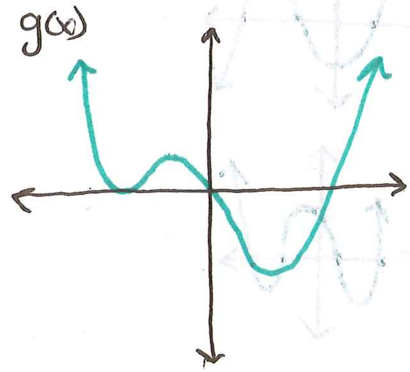
$f(x) = x^4 - 4x^3 - 4x^2 + 16x$ $g(x) = x^4 + x^3 - 8x^2 - 12x$ $h(x) = x^4 - 4x^3 + 16x - 16$

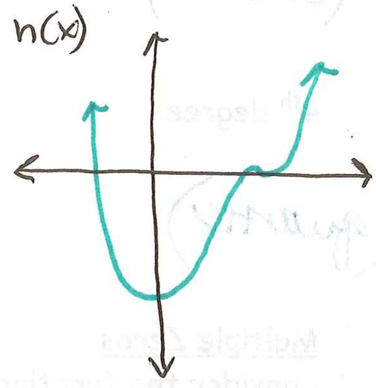
On a large scale they all resemble the graph of ...



On a smaller scale, the graphs look like...







Zeros of a Polynomial Function

The zeros of a function are the values of __x__ that make __y__ equal zero (that is, the values of __x__ that make the __function__ equal zero).

The zeros of a function are also sometimes referred to as:

roots, solutions, x-intercepts

The total number of zeros that a polynomial function has is always equal to the __degree__ of the polynomial. Also note that every __zero__ corresponds to a __factor__. For example, if x = 2 is a __zero__ then x-2 is a __factor__.

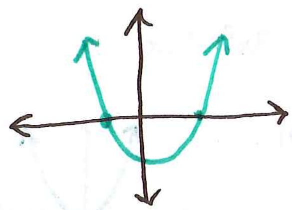
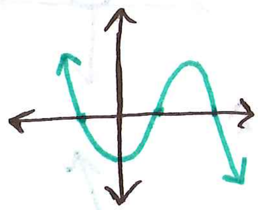
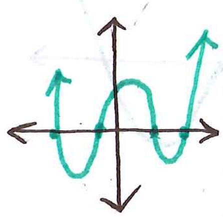
$f(x) = x^2 + 5x - 14$

$(x-2)(x+7)$ $x = \overbrace{2, -7}^{\text{zeros}}$

Bumps/Turns

Polynomial functions also have another characteristic that we refer to as bumps or turns. A __bump/turn__ corresponds to a change in direction for the graph of a function. Between any two consecutive __zeros__, there must be a __bump/turn__ because the graph would have to change direction or turn in order to cross the x-axis again.

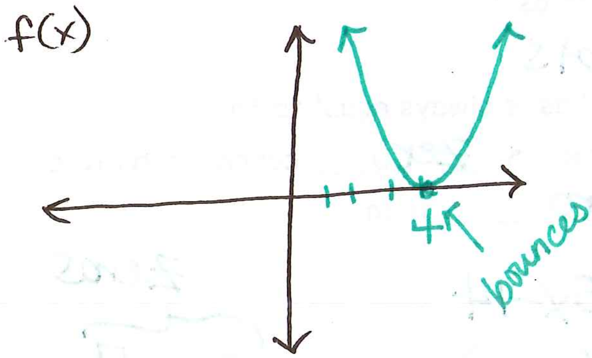
How does the number of bumps/turns compare to the number of zeros?

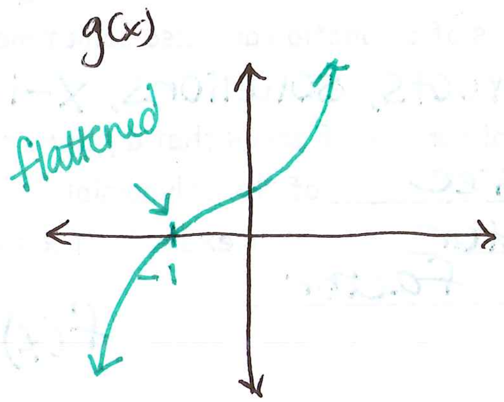
Type of polynomial	Typical graph shape	# of zeros	# of bumps
2^{nd} degree (quadratic)		2	1
3^{rd} degree (cubic)		3	2
4^{th} degree (quartic)		4	3

# of bumps is always one less than the # of zeros

Multiple Zeros

Consider the functions $f(x) = x^2 - 8x + 16$ and $g(x) = x^3 + 3x^2 + 3x + 1$. How many zeros should f(x) have? 2 What about g(x)? 3 Now look at the graphs on your calculator and determine what the actual zeros are.



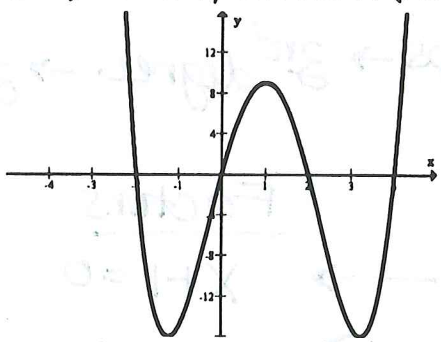


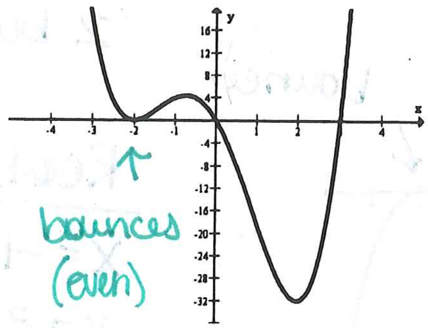
For f(x) we say that x= 4 is a double root and for g(x) we say that x = -1 is a triple root.

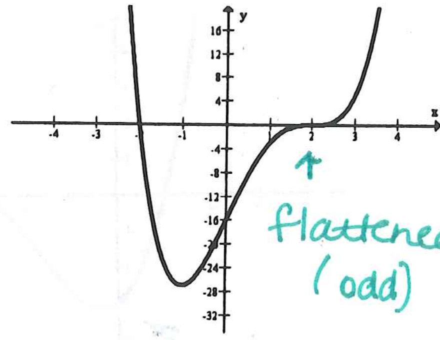
If the graph of a polynomial function bounces off the x-axis, then the zero at that point will be repeated an even number of times.

If the graph of a polynomial function crosses the x-axis but looks flattened there, then the zero at that point will be repeated an odd number of times.

Ex 1) Identify the zeros (roots) and factors of the following polynomials.







4^{th} degree, 4 roots

Roots

$X = -2$

$X = 0$

$X = 2$

$X = 4$

4^{th} degree, 4 roots

Roots

$X = -2$

$X = -2$

$X = 0$

$X = 3$

4^{th} degree: 4 roots

Roots

$X = -2$

$X = 2$

$X = 2$

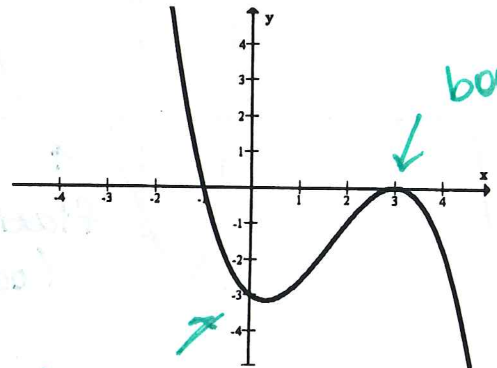
$X = 2$

Polynomials in Factored Form

If we know the zeros of a polynomial function then we can write the polynomial in factored form, and can use the factored form to come up with a formula for the polynomial.

Factored Form for a polynomial is f(x) = $a(x-r_1)(x-r_2)(x-r_3)\ldots$ (and so on depending on the number of factors) where a is a constant and the "r's" are the roots (or zeros).

Ex. 2) Find a formula for this polynomial.



bounce

2 bumps → 3rd degree → 3 roots

y-intercept
(0, -3)
use for formula

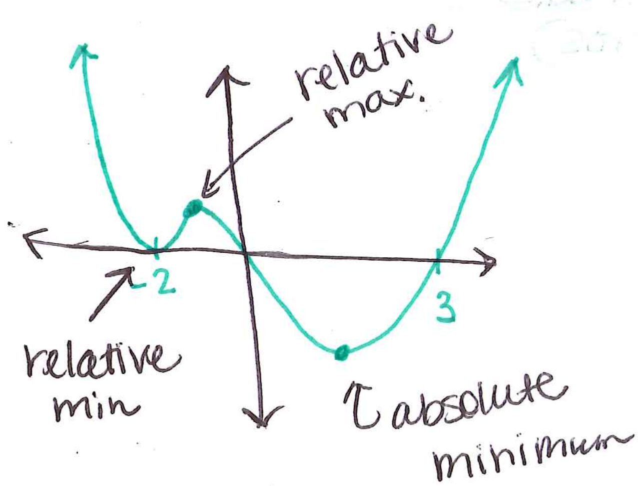
Roots		Factors
$x=-1$	→	$x+1=0$
$x=3$	→	$x-3=0$
$x=3$	→	$x-3=0$

$$\boxed{y=-\frac{1}{3}(x+1)(x-3)(x-3)}$$

$$y=a(x+1)(x-3)(x-3)$$
$$-3=a(0+1)(0-3)(0-3)$$
$$-3=a(1)(-3)(-3)$$
$$\frac{-3}{9}=\frac{a\cdot 9}{9} \qquad a=\frac{-3}{9}=\frac{-1}{3}$$

Maximum and Minimum Values of a Polynomial Function
Another important characteristic of polynomial functions is their maximum or minimum y-values. Polynomials may have a relative maximum or a relative minimum, which means a maximum or minimum value within a particular range of x-values. They may also have an absolute maximum or an absolute minimum, which means a y-value that is a maximum or minimum over the entire domain of the polynomial.

Ex 2) Sketch a graph of $h(x) = x^4 + x^3 - 8x^2 - 12x$ and identify any maximum or minimum values.



#5-9

Name *Key* Class________ Date 3/17/15 HW

3-1 Practice

Form K

Polynomials, Linear Factors, and Zeros

Write each polynomial in factored form. Check by multiplication.

1. $x^3 + 11x^2 + 30x$

To start, factor out the GCF, x. $x(x^2 + 11x + 30)$

$x(x+5)(x+6)$

2. $x^3 - 3x^2 - x + 3$

$x^2(x-3) - 1(x-3)$

$(x^2-1)(x-3)$

3. $x^2 - 4x - 12$

$(x-6)(x+2)$

4. $x^3 - 81x$

$x(x^2-81)$

$x(x+9)(x-9)$

5. $x^3 + 9x^2 + 18x$

$x(x+9x+18)$

$x(x+6)(x+3)$

Find the zeros of each function. Then graph the function.

6. $y = (x + 2)(x + 3)$

$x = -3, -2$

7. $y = x(x - 1)(x + 3)$

$x = 0$

$x = 1$ $x = -3$

8. $y = (x - 4)(x - 1)$

$x = 4, 1$

9. $y = x(x - 5)(x + 2)$

$x = 0, x = 5, x = -2$

Write a polynomial function in standard form with the given zeros.

10. $x = -2, 1, 4$

To start, write a linear factor for each zero. $(x - (-2))(x - 1)(x - 4)$

Simplify $(x + 2)(x - 1)(x - 4)$

$x^2 + 2x - x - 2$

$(x^2 + x - 2)(x-4) = x^3 + 2x - 2x$

$-4x^2 - 4x + 8$

$x^3 - 4x^2 - 4x + 8$

11. $x = 3, 0$

$(x-3)(x)$

$x^2 - 3x$

12. $3, -8, 0$

$(x)(x-3)(x+8)$

$x(x^2 - 3x + 8x - 24)$

$x^3 + 5x^2 - 24x$

13. $x = 3, -2, 1$

$(x-3)(x+2)(x-1)$

$x^2 - 3x + 2x - 6$ $x^3 - x^2 - 6x$

$(x^2 - x - 6)(x-1)$ $-x^2 + x + 6$

$x^3 - 2x^2 - 5x + 6$

14. $x = -4, 1$

$(x+4)(x-1)$

$x^2 + 4x - x - 4$

$x^2 + 3x - 4$

Name____________________ Class__________ Date__________

3-1 Practice (continued)

Form K

Polynomials, Linear Factors, and Zeros

Find the zeros of each function. State the multiplicity of multiple zeros.

15. $y = (x - 3)^2(x + 1)$

$X=3$ m2
$X=-1$ m1

To start, identify the zeros. The zeros are 3 and -1.

$(X-3)(X-3)(X+1)$

$X=-2$ m1
$X=-1$ m1

16. $y = x^2 + 3x + 2$

$(X+2)(X+1)$

17. $y = (x + 5)^2$

$(X+5)(X+5)$

$X=-5$ m2

$Y=9$ m2

18. $y = (x - 9)^2$

$(X-9)(X-9)$

19. $y = 2x^2 - 2x$

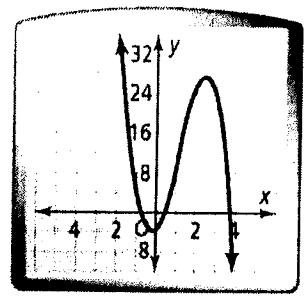
$2x(X-1)$

$X=0$ m1 $X=1$ m1

Find the relative maximum and relative minimum of the graph of each function.

20. $f(x) = -3x^3 + 10x^2 + 6x - 3$

To start, use a graphing calculator.
(An approximate viewing window is
$-5 \le x \le 5$ and $-10 \le y \le 30$.)



max (2.5, 27.6)
min (-0.3, -3.8)

21. $f(x) = x^3 + 4x^2 - x + 1$

max (-2.8, 13.2)
min (0.1, 0.9)

22. $f(x) = x^3 - 6x + 9$

max(-1.4, 14.7)
min (1.4, 3.3)

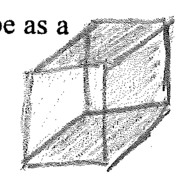
23. Reasoning A polynomial function has a zero at $x = b$. Find one of its factors.

$(X-b)$

cube all sides are the same

24. The side of a cube measures $2x + 1$ units long. Express the volume of the cube as a polynomial.

$V=l \cdot w \cdot h$



2x+1

$(2x+1)(2x+1)(2x+1)$

$8x^3+12x^2+6x+1 = (4x^2+4x+1)(2x+1)$

(h) (L) (w)
$X+2X+W=10$
$W=10-3X$

25. The length of a box is 2 times the height. The sum of the length, width, and height of the box is 10 centimeters.

a. Write expressions for the dimensions of the box. length = 2x height = x width = 10-3x

b. Write a polynomial function for the volume of the box. (To start, write the function in factored form).

c. Find the maximum volume of the box and the dimensions of the box that generates this volume.

$(2x)(x)(10-3x)$
$(2x^2)(10-3x)$
$20x^2-6x^3$

C) max volume: 32.9 (2nd trace max)

Math III 2-2

Name * KEY *

Notes ~~3-2~~ Polynomials, Linear Factors, and Zero

Date________________ Period ______

Finding the Zeros of a polynomial function will help you:

- factor the polynomial
- graph the polynomial
- Solve the related polynomial equation

Writing a Polynomial in Factored Form

What is the factored form of $x^3 - 2x^2 - 15x$?

$x(x^2-2x-15)$	Factor out the GCF, x.
$x(x-5)(x+3)$	Factor $x^2 - 2x - 15$
Check:	
$x(x^2-2x-15)$	Multiply $(x-5)(x+3)$
x^3-2x^2-15x	Distributive Property

Roots, Zeros, and x-intercepts

The following are equivalent statements about a real number *b* and a polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$.

- $x-b$ is a linear factor of polynomial $P(x)$
- b is a zero of the polynomial function $y=P(x)$
- b is a root (or solution) of polynomial $P(x)=0$ equations
- b is an x-intercept of the graph of $y=P(x)$

Finding Zeros of a Polynomial Function

What are the zeros of $y=(x+2)(x-1)(x-3)$?

Use the Zero-Product Property to find the zeros.

$x+2=0$ $x-1=0$ $x-3=0$

$x=-2$ $x=1$ $x=3$

Factor Theorem

The Factor Theorem describes the relationship between the linear factor of a polynomial and the zeros of a polynomial.

Factor Theorem

The expression $x-a$ is a factor of a polynomial if and only if the value a is a zero of the related polynomial function.

Writing a Polynomial Function From Its Zeros

A. What is a cubic polynomial function in standard form with zeros -2, 2, and 3?

$$f(x)=(x+2)(x-2)(x-3)$$

$$=(x^2-4)(x-3)$$

$$=x^3-3x^2-4x+12$$

B. What is a quartic polynomial function in standard form with zeros $-2,\ -2,\ 2,$ and 3?

$$g(x) = \underbrace{(x+2)(x+2)}\underbrace{(x-2)(x-3)}$$

$$(x^2+4x+4)(x^2-5x+6)$$

$$x^4 - 5x^3 + 6x^2 + 4x^3 - 20x^2 + 24x + 4x^2 - 20x + 24$$

$$g(x) = x^4 - x^3 - 10x^2 + 4x + 24$$

Graph both functions.

1. How do the graphs differ?

2. How are they similar?

Multiple Zeros -

when the factors are repeated

* ex: $(x-2)(x-2)(x-2)$

Zero of multiplicity -

x-a appears n times as a factor

* Ex: $(x-2)(x-2)(x-2) \rightarrow$ Zero is 2, multiplicity is 3

Ex: $(x+1)^3 \rightarrow$ zero is -1, multiplicity is 3

How Multiple Zeros Affect a Graph

If *a* is a zero of multiplicity *n* in the polynomial function $y = P(x)$, then the behavior of the graph at the x-intercept *a* will be:

- close to linear if $n=1$
- close to quadratic if $n=2$
- close to cubic if $n=3$
- and so on ...

Finding the Multiplicity of a Zero

What are the zeros of $f(x) = x^4 - 2x^3 - 8x^2$?

$f(x) = x^2(x^2 - 2x - 8)$

$= x^2(x-4)(x+2)$ → Zeros: $0, 0, 4, -2$

What are their multiplicities?

0: m of 2 4: m of 1 -2: m of 1

How does the graph behave at these zeros?

At 0 → quadratic

At 4 → linear At -2 → linear

2-3 Notes

Notes

2/11/15

CC3 Unit 2, Polynomial & Rational Functions **Name:** Key

Notes ~~2-2~~ Real and Imaginary Numbers Date: ______ **Period:** ______

Deriving the Quadratic Formula

$ax^2 + bx + c = 0$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$	Divide each side by *a*.
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	Rewrite so all terms containing *x* are on one side.
$x^2 + \frac{b}{a}x + \left(\frac{b}{a}\right)^2 = \left(\frac{b}{a}\right)^2 - \frac{c}{a}$	Complete the Square.
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	Factor the perfect square trinomial. Also simplify.
$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$	Find square roots.
$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$	Solve for *x*. Also simplify the radical.
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Simplify.

The Discriminant

$ax^2 + bx + c = 0$

where a, b, c are real #s

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac$

- is greater than zero, then 2 real solutions
- is equal to zero, then 1 real solution
- is less than zero, then imaginary solutions / no real (non-real)

Non-real solutions to the quadratic formula are known as Imaginary numbers.

Real numbers and Imaginary numbers are a subset of a larger set of numbers known as Complex numbers.

Essential Understanding

The complex numbers are based on a number whose square is -1.

The imaginary unit is the complex number whose square is -1. So, $i^2 = -1$, and $i = \sqrt{-1}$.

Square Root of a Negative Real Number

For any positive number a, $\sqrt{-a} = \sqrt{-1 \cdot a} = \sqrt{-1} \cdot \sqrt{a} = i\sqrt{a}.$

$\sqrt{-5} = \pm i\sqrt{5}$ Note that $(\sqrt{-5})^2 = (i\sqrt{5})^2 = i^2(\sqrt{5})^2 = -1 \cdot 5 = -5$ (not 5).

Simplify a Number Using *i*

How do you write $\sqrt{-18}$ by using the imaginary unit i?

$\sqrt{-1} \cdot \sqrt{18}$	Multiplication Property of Square Roots
$i \cdot \sqrt{18}$	Definition of i
$i \; 3\sqrt{2}$	Simplify.
$3i\sqrt{2}$	

18
^
2 9 ← pefect square

An imaginary # is any number of the form $a + bi$, where a and b are real numbers and $b \neq 0$.

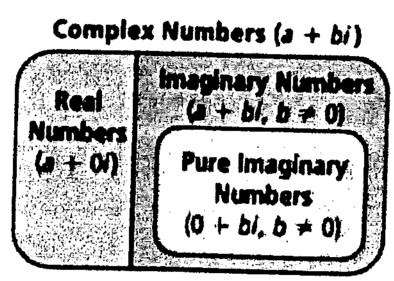
take note **Key Concept Complex Numbers**

You can write a **complex number** in the form $a + bi$, where a and b are real numbers.

If $b = 0$, the number $a + bi$ is a real number.

If $a = 0$ and $b \neq 0$, the number $a + bi$ is a **pure imaginary number**.

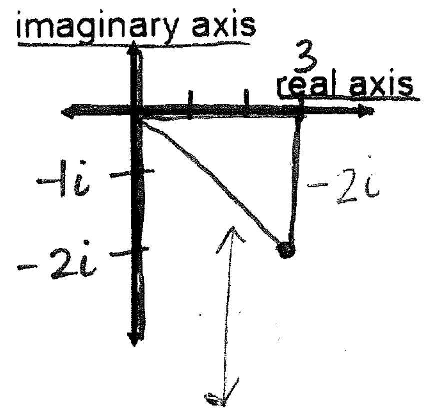
$\underset{\text{Real part}}{a} + \underset{\text{Imaginary part}}{bi}$



Name: KEY

Notes 2-3 Imaginary Numbers (contd) **Date:** ______ **Period:** ______

Complex Number Plane



$|3-2i| = \sqrt{13}$

In the complex number plane, the point (a, b) represents the complex number $a+bi$. To graph a complex number, locate the real part on the horizontal axis and the imaginary part on the vertical axis.

The absolute value of a complex number is its distance from the origin in the complex plane.

$$|a+bi| = \sqrt{a^2+b^2}$$

Graphing in the Complex Number Plane

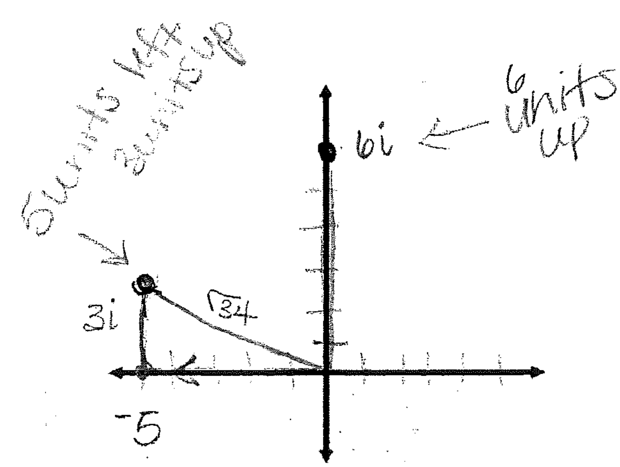
What are the graph and absolute value of each number?

A. $-5+3i$

$$|-5+3i| = \sqrt{(-5)^2+3^2}$$
$$= \sqrt{34}$$

B. $6i$

$$= |0+6i|$$
$$= \sqrt{0^2+6^2}$$
$$= \sqrt{36}$$
$$= 6$$



Adding and Subtracting Complex Numbers

To add or subtract complex numbers, combine the real parts and the imaginary parts separately. The associative and commutative properties apply to complex numbers.

What is each sum or difference?

A. $(4-3i)+(-4+3i)$

$$4-3i-4+3i$$
$$4-4-3i+3i$$
$$0$$

B. $(5-3i)-(-2+4i)$

$$5-3i+2-4i$$
$$5+2-3i-4i$$
$$7-7i$$

Multiplying Complex Numbers

You multiply complex numbers $a+bi$ and $c+di$ as you would multiply binomials. For imaginary parts bi and di, $(bi)(di)=bd(i)^2=bd(-1)=-bd$.

Example: What is each product?

A. $(3i)(-5+2i)$	$-15i+6i^2$ $-15i+6(-1)$ $-15i-6$
B. $(4+3i)(-1-2i)$	$4-8i+3i-6i^2$ $4-5i-6(-1)$ $4-5i+6$ $10+5i$
C. $(-6+i)(-6-i)$	$+36+6i-6i-i^2$ $36-(-1)$ $36+1 = 37$ The solution to this problem is a real number.

Number pairs of the form $a+bi$ and $a-bi$ are complex conjugates.

The product of these types of pairs is a real number.

$$(a+bi)(a-bi)=a^2-(bi)^2=a^2-b^2i^2=a^2-b^2(-1)=a^2+b^2$$

Dividing Complex Numbers

You can use complex conjugates to simplify quotients of complex numbers.

What is each quotient?

A. $\frac{9+12i}{3i}\cdot\frac{-3i}{-3i}=\frac{-27i-36i^2}{-9i^2}$ $=\frac{-27i+36}{-9(-1)}=\frac{-27i+36}{9}$	B. $\frac{2+3i}{1-4i}=\frac{1+4i}{1+4i}=\frac{2+8i+3i+12i^2}{1+4i-4i-16i^2}$ $=\frac{2+11i+12(-1)}{1-16(-1)}$ $=\frac{-10+11i}{17}$

$\frac{-27i}{9}+\frac{36}{9}$

$-3i+4$

$=\frac{-10}{17}+\frac{11i}{17}$

Finding Pure Imaginary Solutions

What are the solutions of $2x^2 + 32 = 0$?

$$\frac{2x^2}{2} = \frac{-32}{2}$$

$$x^2 = -16$$

$$x = \pm\sqrt{-16}$$

$$x = \pm 4i$$

$$\sqrt{-1 \cdot 16}$$

$$\sqrt{-1} \cdot \sqrt{16}$$

$$i \quad \pm 4$$

$$\pm 4i$$

Finding Imaginary Solutions

What are the solutions of $2x^2 - 3x + 5 = 0$? Use quadratic formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(5)}}{2(2)} = \frac{3 \pm \sqrt{9 - 40}}{4}$$

$$= \frac{3 \pm \sqrt{-31}}{4}$$

$$= \frac{3}{4} \pm \frac{i\sqrt{31}}{4}$$

CC3 Unit 2, Polynomial & Rational Functions — Name: Key

Notes ~~2.1~~ 2.4 Quadratic Equations — Date: ______ Period: 3rd

Solving Quadratic Equations Review

Some quadratic equations can be solved by factoring.
Others can be solved just by using square roots.
ANY quadratic equation can be solved by using quadratic formula.

Solving by Finding Square Roots

a. $4x^2 + 10 = 46$

$-10 \quad -10$

$4x^2 = 36$

$\frac{4x^2}{4} = \frac{36}{4}$

$x^2 = 9$

$x = \pm 3$

b. $3x^2 - 5 = 25$

$+5 \quad +5$

$3x^2 = 30$

$\frac{3x^2}{3} = \frac{30}{3}$

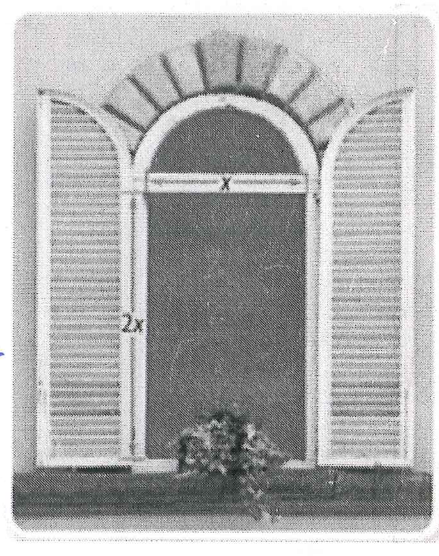
$x^2 = 10$

$x = \pm\sqrt{10}$

Determining Dimensions

While designing a house, an architect used windows like the one shown here. What are the dimensions of the window if it has 2766 square inches of glass?

r = radius



Step 1: Find the area of the window

(rectangle) $A = l \cdot w \quad (2x)(x) = 2x^2$ in

(semi-circle) $= \frac{1}{2}\pi r^2$

$\frac{1}{2}\pi\left(\frac{x}{2}\right)^2 = \frac{1}{2}\pi\frac{x^2}{4} = \frac{\pi}{8}x^2 \text{ in}^2$

Total Area $\Rightarrow 2x^2 + \frac{\pi}{8}x^2 = 2766$

Solve for x: $x^2\left(2 + \frac{\pi}{8}\right) = 2766 \Rightarrow x^2 = \frac{2766}{2 + \frac{\pi}{8}}$ $\quad x = \pm 34$

Length can't be negative. So $L = 34 \quad W = 2(34) = 68 \quad R = r = \frac{34}{2}$

Solving a Perfect Square Trinomial Equation

What is the solution of $x^2 + 4x + 4 = 25$?

Factor the Perfect Square Trinomial

$x^2 + 4x + 4 = 25$
$(x+2)^2 = 25$
$x + 2 = \pm 5$

Find Square Roots

Rewrite as two equations

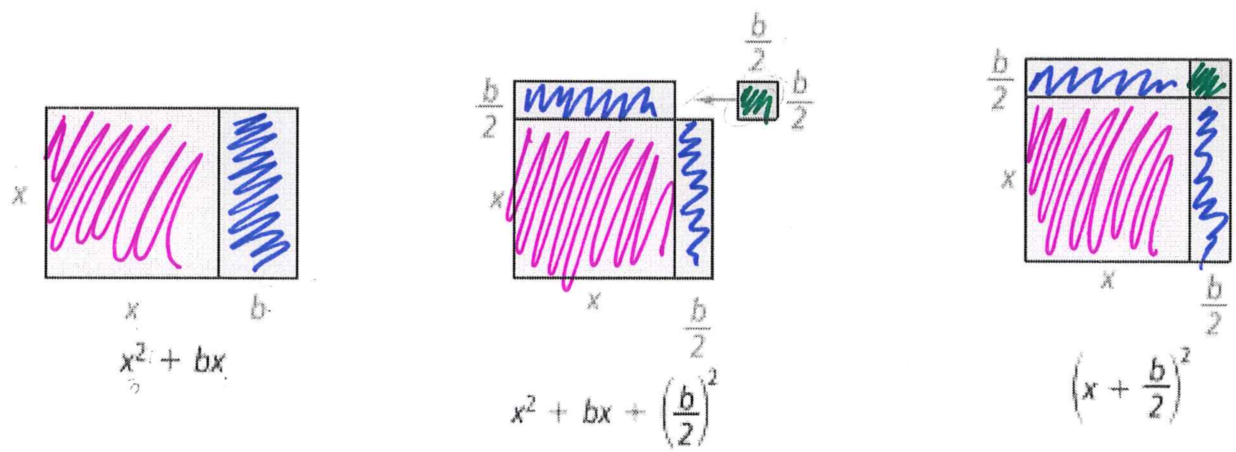
$x + 2 = -5 \quad x + 2 = 5$

Solve for x

$x = -7 \;, \quad x = 3$

Completing the Square

If $x^2 + bx$ is not part of a perfect square trinomial, you can use the coefficient b to find a constant c so that $x^2 + bx + c$ is a perfect square. When you do this, you are completing the square. The diagram models this process.



You can form a perfect square trinomial from $x^2 + bx$ by adding $\left(\frac{b}{2}\right)^2$.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Example: What value completes the square for $x^2 - 10x$? Justify your answer.

$x^2 - 10x$ identify b; $b = -10$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-10}{2}\right)^2 = (-5)^2 = 25$ Find $\left(\frac{b}{2}\right)^2$

$x^2 - 10x + 25$ Add the values of $\left(\frac{b}{2}\right)^2$ to complete the square.

$x^2 - 10x + 25 = (x-5)^2$ Rewrite as the square of a binomial

Solving an Equation by Completing the Square

1. Rewrite the equation in the form $x^2 + bx = c$. To do this, get all terms with the variable on one side of the equation and the constant on the other side. Divide all the terms of the equation by the coefficient of x^2 if it is not 1.
2. Complete the square by adding $\left(\frac{b}{2}\right)^2$ to each side of the equation.
3. Factor the trinomial.
4. Find square roots.
5. Solve for *x*.

Example 1 – What is the solution of $3x^2 - 12x + 6 = 0$?

$-6 \quad -6$

$3x^2 - 12x = -6$

$\frac{3x^2 - 12x}{3} = \frac{-6}{3}$

$\left(\frac{-4}{2}\right)^2 = (2)^2 = 4$

$x^2 - 4x = -2$

$x^2 - 4x + 4 = -2 + 4$

$(x-2)^2 = 2$

$x - 2 = \pm\sqrt{2}$

$x - 2 = +\sqrt{2}$ $\quad$ $x - 2 = -\sqrt{2}$

$x = 2 + \sqrt{2}$ $\quad$ $x = 2 - \sqrt{2}$

Example 2 – What is the solution of $2x^2 - x + 3 = x + 9$?

$$2x^2 - x = x + 6$$
$$-x \quad -x$$
$$2x^2 - 2x = 6$$

$$\frac{2x^2 - 2x}{2} = \frac{6}{2}$$

$$x^2 - x = 3$$

$$\left(-\frac{1}{2}\right)^2 = .25$$
or $1/4$

$$x^2 - x + \frac{1}{4} = 3 + \frac{1}{4}$$

$$(x - 1/2)^2 = 3.25$$

$$x = \frac{1}{2} \pm \sqrt{3.25}$$

Writing in Vertex Form

What is $y = x^2 + 4x - 6$ in vertex form? Name the vertex and y-intercept.

$$y = a(x-h)^2 + k$$

y-intercept

$$y = x^2 + 4x - 6$$

$a = 1$

$c = -6$

$$x^2 + 4x = +6$$
$$\left(\frac{4}{2}\right)^2 = 4$$
$$x^2 + 4x + 4 = +6 + 4$$
$$(x+2)^2 = +10$$

$$y = 1(x+2)^2 - 10$$

$$y = (x + \underline{2})^2 \underline{-10}$$

vertex (h, k) → h k

$(-2, -10)$

(change the sign)

$y = (0, -6)$

12) $x^2 + \frac{1}{2}x + 1/16 = 1$

$(x + 1/4)^2 = 1$

$x = \pm 1 - 1/4$

$x = -5/4, \frac{3}{4}$

35) $4x^2 - kx + 9 = 0$

$x^2 - \frac{kx}{4} + \frac{9}{4} = 0$

$\left(\frac{\frac{b}{4}}{2}\right)^2 = \frac{9}{4}$

$\left(\frac{\frac{b}{4}}{2}\right) = \frac{3}{2}$

$\frac{b}{4} = \frac{6}{2}$

$b = \frac{24}{2}$

$\boxed{b = 12}$

36) $16x^2 + kx + 9 = 0$

$\left(\frac{\frac{b}{16}}{2}\right)^2 = \frac{9}{16}$

$\frac{\frac{b}{16}}{2} = \frac{3}{4}$

$\frac{b}{16} = \frac{6}{4}$

$\boxed{b = 24}$

18) $25x^2 + 10x + \boxed{}$

$\left(\frac{\frac{10}{25}}{2}\right)^2 = (0.4) * 25 = 1$

OR $\left(\frac{10}{2}\right)^2 = \frac{25}{25} = 1$

Name Key Class ____________ Date____________

1-4 Practice

Completing the Square

Solve each equation by finding square roots. To start, remember to isolate x^2.

1. $x^2 - 9 = 0$

$x^2 = 9$

2. $x^2 + 4 = 20$

$x^2 = 16$

3. $x^2 + 15 = 16$

$x^2 = 1$

4. $2x^2 - 64 = 0$

5. $4x^2 - 100 = 0$

6. $5x^2 - 25 = 0$

7. You are painting a large wall mural. The wall length is 3 times the height. The area of the wall is 300 ft^2.

a. What are the dimensions of the wall? $10 \cdot 30$

b. If each can of paint covers 22 ft^2, will 12 cans be enough to cover the wall?

7

$W = 3h$

$a = l \cdot w$

$300 = 3h \cdot h$

$300 = 3h^2$

$100 = h^2$

$10 = h$

x^2 $x \cdot$

8. The lengths of the sides of a carpet have the ratio of 4.4 to 1. The area of the carpet is 1154.7 ft^2. What are the dimensions of the carpet?

8

$l \cdot w$

$4.4 \cdot 1$

$a = l \cdot w$

$1154.7 = \frac{4.4}{1}$

$1154.7 = 4.4w \cdot w$

$1154.7 = 4.4w^2$

9. A packing box is 4 ft deep. One side of the box is 1.5 times longer than the other. The volume of the box is 24 ft^3. What are the dimensions of the box?

$24 = (4)(x)(1.5x)$

$24 = 6x^2$

$x = 2$

2, 4, 3

Solve each equation. To start, factor the perfect square trinomial.

10. $x^2 - 14x + 49 = 81$

$(x - 7)^2 = 81$

11. $x^2 + 6x + 9 = 1$

$(x + 3)^2 = 1$

12. $9x^2 - 12x + 4 = 49$

$(3x - 2)^2 = 49$

13. $4x^2 + 36x + 81 = 16$

14. $x^2 + 2x + 1 = 36$

15. $x^2 - 16x + 64 = 9$

Name________________________________ Class______________ Date____________

1-4 Practice (continued)

Completing the Square

Complete the following squares.

16. $x^2 + 8x + \boxed{}$

$\left(\frac{8}{2}\right)^2 = 4^2 = 16$

17. $x^2 + 20x + \boxed{}$

$\left(\frac{20}{2}\right)^2 =$

18. $x^2 - 14x + \boxed{}$

19. $x^2 - 24x + \boxed{}$

20. $x^2 + 34x + \boxed{}$

21. $x^2 - 46x + \boxed{}$

Solve the following equations by completing the square.

22. $x^2 - 8x - 5 = 0$

$x^2 - 8x = 5$

$x^2 - 8x + 16 = 5 + 16$

$(x - 4)^2 = 21$

$x - 4 = \pm\sqrt{21}$

$x = \boxed{}$

23. $x^2 + 12x + 9 = 0$

$x^2 + 12x = -9$

$x^2 + 12x + 36 = -9 + 36$

24. $x^2 - 10x = -11$

25. $2x^2 + 11x - 23 = -x + 3$

26. $x^2 - 18x + 64 = 0$

27. $3x^2 - 42x + 78 = 0$

Write the following equations in vertex form.

28. $y = x^2 + 10x - 9$

29. $y = x^2 - 18x + 13$

30. $y = x^2 + 32x - 8$

Math III ~~3-3~~ 2-5

Notes ~~3-3~~ Solving Polynomials Equations

Name * Key *

Date________ Period______

To solve a polynomial equation by factoring:

1. Write the equation in the form $P(x)=0$ for some polynomial function P.
2. Factor $P(x)$. Use the Zero Product Property to find the Roots.

Solving Polynomial Equations Using Factors

What are the real or imaginary solutions of each polynomial equation?

A. $2x^3 - 5x^2 = 3x$

$2x^3 - 5x^2 - 3x = 0$	Rewrite in the form P(x) = 0.
$x(2x^2 - 5x - 3) = 0$	Factor out the GCF, x.
$x(2x+1)(x-3) = 0$	Factor $2x^3 - 5x^2 = 3x$
$x=0 \quad 2x+1=0 \quad x-3=0$	Zero Product Property
$x=0 \quad x=-1/2 \quad x=3$	Solve each equation for x.
The solutions are $0, -1/2, 3$	

B. $3x^4 + 12x^2 = 6x^3$

$3x^4 - 6x^3 + 12x^2 = 0$	Rewrite in the form P(x) = 0.
$x^4 - 2x^3 + 4x^2 = 0$	Multiply by $\frac{1}{3}$ to simplify
$x^2(x^2 - 2x + 4)$	Factor out the GCF, x^2
$x^2 = 0 \quad x^2 - 2x + 4 = 0$	Zero Product Property
$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(4)(1)}}{2(1)}$ $= \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm 2i\sqrt{3}}{2} =$	Use the Quadratic Formula $1 \pm i\sqrt{3}$
The solutions are $0, 1+i\sqrt{3}, 1-i\sqrt{3}$	

Polynomial Factoring Techniques	
Techniques	**Examples**
Factoring out the GCF Factor out the greatest common factor of all three terms	$15x^4 - 20x^3 + 35x^2$ $5x^2(3x^2 - 4x + 7)$
Quadratic Trinomials Factor ax^2+bx+c, find factors with products $a \cdot c$ and sum b	$6x^2+11x-10$ $(3x-2)(2x+5)$
Perfect Square Trinomials $a^2 + 2ab + b^2 = (a+b)^2$ $a^2 - 2ab + b^2 = (a-b)^2$	$x^2+10x+25=(x+5)^2$ $x^2-10x+25 = (x-5)^2$
Difference of Squares $a^2-b^2=(a+b)(a-b)$	$4x^2-15 =$ $(2x+\sqrt{15})(2x-\sqrt{15})$
Factor by Grouping $ax+ay+bx+by$ $= a(x+y) + b(x+y)$ $= (a+b)(x+y)$	x^3+2x^2-3x-6 $x^2(x+2) + (-3)(x+2)$ $(x^2-3)(x+2)$
Sum of Difference of Cubes $a^3+b^3=(a+b)(a^2-ab+b^2)$ $a^3-b^3=(a-b)(a^2+ab+b^2)$	$8x^3+1=(2x+1)(4x^2-2x+1)$ $8x^3-1=(2x-1)(4x^2+2x+1)$

The sum and difference of cubes is a new factoring technique.

(SKIP)

Why it Works

Pick up here

$a^3+b^3=(a+b)(a^2-ab+b^2)$ $ab^2 \times b^3$

$a^3+b^3=a^3+a^2b-a^2b+ab^2$	Add 0.
$=a^2(a+b)-ab(a+b)-b^2(a+b)$	Factor out a^2, $-ab$, and b^2.
$=(a+b)(a^2-ab-b^2)$	Factor out $(a+b)$.

Solving Polynomial Equations by factoring

What are the real and imaginary solutions of each polynomial equation?

A. $x^4-3x^2=4$

$x^4-3x^2-4=0$	Rewrite in the form P(x) = 0.
$a^2-3a-4=0$	Let a = x^2.
$(a-4)(a+1)=0$	Factor.
$(x^2-4)(x^2+1)=0$	Replace a with x^2.
$(x+2)(x-2)(x^2+1)$	Factor x^2-4 as a difference of squares
zeros: $x=2,-2$; $x^2=-1 \rightarrow x=\sqrt{-1} \rightarrow x=\pm i$ — zeros: $2,-2,i,-i$	

B. $x^3=1$

$x^3-1=0$	Rewrite in the form P(x) = 0.
$(x-1)(x^2+x+1)$	Factor the difference of cubes.
$x=\dfrac{-1\pm\sqrt{1^2-(4\cdot1\cdot1)}}{2\cdot1}=\dfrac{-1\pm\sqrt{-3}}{2}=\dfrac{-1\pm i\sqrt{3}}{2}$ Solutions: $x=1,\ -\dfrac{1+i\sqrt{3}}{2},\ \dfrac{-1-i\sqrt{3}}{2}$	

4"

Your Turn

1. $x^4 = 16$

$x^4 - 16 = 0$

$a^2 - 16 = 0$

$(a+4)(a-4)$

$(x^2+4)(x^2-4)$

$x^2 = -4$

$x = \sqrt{-4}$

$x = \pm i2$

$x^2 = 4$

$x = \pm 2$

zeros, $2, -2, -2i, 2i$

2. $x^3 = 8x - 2x^2$

$x^3 - 8x + 2x^2 = 0$

$x(x^2 + 2x - 8) = 0$

$x(x+4)(x-2) = 0$

$x = 0, -4, 2$

3. $x(x^2 + 8) = 8(x+1)$

$x^3 + 8x = 8x + 8$

$x^3 + 8x - 8x - 8 = 0$

$x^3 - 8 = 0$

$(x-2)\underbrace{(x^2+2x+4)}_{\text{quad. formula}}$

$x = 2$

$x = \dfrac{-2 \pm \sqrt{4 - (4 \cdot 1 \cdot 4)}}{2}$

$x = \dfrac{-2 \pm \sqrt{-12}}{2}$

$= \dfrac{-2 \pm 2i\sqrt{3}}{2} = -1 \pm i\sqrt{3}$

Finding Real Roots by Graphing

What are the real solutions of the equation $x^3 + 5 = 4x^2 + x$?

Use **INTERSECT** feature	Use **ZERO** feature
Set $y_1 = x^3+5$ & $y_2 = 4x^2+x$	Rewrite as x^3-4x^2-x+5
use Intersect	Put in $y_1 =$ and graph
Appoximate Points of	use zero feature to find
Intersection	x-intercept

Math III 2-6

Notes ~~3-4~~ Dividing Polynomials

Name * KEY

Date________________ Period ______

Review - Operations with Polynomials

Addition of Polynomials

The sum of two polynomials is found by combining like terms. To add like terms, add the coefficients and do not change the variable and exponents in common.

Examples

A. $(2x^7+9x^3-5)+(3x^3+2x+14)$

$2x^7+9x^3-5+3x^3+2x+14$

$2x^7+12x^3+2x+9$

B. $(2x^3+17x^2-5x)+(3x^3-x+8)$

$2x^3+17x^2-5x+3x^3-x+8$

$5x^3+17x^2-6x+8$

Subtraction of Polynomials

The difference of two polynomials is found by adding the first polynomial to the negative of the second polynomial. The negative of the second polynomial is found by changing the sign of each term of the polynomial.

Examples

A. $(9x^5+2x^3-1)-(2x^3+4x-4)$

$9x^5+2x^3-1-2x^3-4x+4$

$9x^5-4x+3$

B. $(5x^2-7x+2)-(x^2+4x-3)$

$5x^2-7x+2-x^2-4x+3$

$4x^2-11x+5$

Math III Notes 3-4 Dividing Polynomials

Multiplication of Polynomials
Multiplication of polynomials is done by repeated use of the distributed property. Multiplication of binomials (two-termed polynomials) is done using the **FOIL** method. FOIL is an acronym which stands for "first terms, outside terms, inside terms, last terms."

Examples
A. $(-7x+2)(4x-3)$

$-28x^2+21x+8x-6$

$-28x^2+29x-6$

B. $(9x^3-5)(3x^3+2x)$

$27x^6+18x^4-15x^3-10x$

C. $(2x^3+9x^2-5)(3x^2+2x+14)$

	$2x^3$	$9x^2$	-5
$3x^2$	$6x^5$	$27x^4$	$-15x^2$
$2x$	$4x^4$	$18x^3$	$-10x$
14	$28x^3$	$126x^2$	-70

$6x^5+31x^4+46x^3+111x^2-10x-70$

Long Division is one of many methods you can use to divide whole numbers.

You can divide polynomials using steps that are similar to the long-division steps that you use to divide whole numbers.

EXAMPLES

Numerical Long Division

$$\begin{array}{r} 32 \\ 21\overline{)672} \\ -63\downarrow \\ \hline 42 \\ -42 \\ \hline 0 \end{array}$$

Polynomial Long Division

$$\begin{array}{r} 3x+2 \\ 2x+1\overline{)6x^2+7x+2} \\ \cancel{6x^2}+3x\downarrow \\ \hline 4x+2 \\ -4x+2 \\ \hline 0 \end{array}$$

The remainder from each division above is 0, so 21 is a factor of 672 and 2x+1 is a factor of $6x^2+7x+2$.

Using Polynomial Long Division

Use polynomial long division to divide $4x^2+23x-16$ by $x+5$. What is the quotient and remainder?

$$\begin{array}{r} 4x+3 \; r^-31 \\ x+5\overline{)4x^2+23x-16} \\ 4x^2+20x\downarrow \\ \hline 3x-16 \\ -3x+15 \\ \hline -31 \end{array}$$

Math III Notes 3-4 Dividing Polynomials

Use polynomial long division to divide $3x^2 - 29x + 56$ by $x - 7$. What is the quotient and remainder?

3x−8 R 0

$$x-7\overline{)3x^2-29x+56}$$

3x²−21x ↓

−8x+56

−(−8x+56)

0

(3x−8)(x−7)

take note

Key Concept **The Division Algorithm for Polynomials**

You can divide polynomial $P(x)$ by polynomial $D(x)$ to get polynomial quotient $Q(x)$ and polynomial remainder $R(x)$. The result is $P(x) = D(x)Q(x) + R(x)$.

$$D(x)\overline{)P(x)}^{\;Q(x)}$$

$\vdots$

$R(x)$

If $R(x) = 0$, then $P(x) = D(x)Q(x)$ and $D(x)$ and $Q(x)$ are factors of $P(x)$.

To use long division, $P(x)$ and $D(x)$ should be in standard form with zero coefficients where appropriate. The process stops when the degree of the remainder, $R(x)$, is less than the degree of the divisor, $D(x)$.

Checking Factors

Is $x^2 + 1$ a factor of $3x^4 - 4x^3 + 12x^2 + 5$?

3x²−4x+9 R−4

$$x^2+0x+1\overline{)3x^4-4x^3+12x^2+0x+5}$$

3x⁴−0x³+3x² ↓

−4x³+9x²+0x

−(−4x³+0x²−4x) ↓

9x²+4x+5

−(9x²+0x+9)

−4

Since the remainder is not 0, $x^2 + 1$ is not a factor of $3x^4 - 4x^3 + 12x^2 + 5$.

Math III Notes 3-4 Dividing Polynomials

Synthetic Division

Synthetic Division simplifies the long-division process for dividing by a linear expression $x - a$.

Steps for Synthetic Division:

1. Write the coefficients (including zeros) of the polynomial in standard form.
2. Omit all variables and exponents.
3. For the divisor, reverse the sign (use *a*).
4. This allows you to add instead of subtract throughout the process.

Use synthetic division to divide $x^3 - 14x^2 + 51x - 54$ by $x + 2$. What is the quotient and remainder?

Step 1 - Reverse the sign of +2. Write the coefficients of the polynomial.

$$\underline{-2|}\ \ 1 \quad -14 \quad 51 \quad -54$$

Step 2 - Bring down the first coefficient.

$$\begin{array}{r|rrrr} -2 & 1 & -14 & 51 & -54 \\ & \downarrow & & & \\ \hline & 1 & & & \end{array}$$

Step 3 - Multiply the coefficient by the divisor. Add to the next coefficient.

$$\begin{array}{r|rrrr} -2 & 1 & -14 & 51 & -54 \\ & \downarrow & -2 & & \\ \hline & 1 & -16 & & \end{array}$$

Step 4 - Continue multiplying and adding through the last coefficient.

$$\begin{array}{r|rrrr} -2 & 1 & -14 & 51 & -54 \\ & \downarrow & -2 & 32 & -166 \\ \hline & 1 & -16 & 83 & \boxed{-220} \end{array}$$

The quotient is $x^2 - 16x + 83$, R -220.

Math III Notes 3-4 Dividing Polynomials

Use synthetic division to divide $x^3 - 57x + 56$ by $x - 7$.

$$\begin{array}{r|rrrr} 7 & 1 & 0 & -57 & 56 \\ & \downarrow & 7 & 49 & -56 \\ \hline & 1 & 7 & -8 & \boxed{0} \end{array}$$

$x^2 + 7x - 8$

The Remainder Theorem

If you divide a polynomial $P(x)$ of degree $n \geq 1$ by $x - a$, then the remainder is $P(a)$.

Evaluating a Polynomial

Given that $P(x) = x^5 - 2x^3 - x^2 + 2$, what is $P(3)$?

$$\begin{array}{r|rrrrrr} 3 & 1 & 0 & -2 & -1 & 0 & 2 \\ & \downarrow & 3 & 9 & 21 & 60 & 180 \\ \hline & 1 & 3 & 7 & 20 & 60 & \boxed{182} \end{array}$$

$x^4 + 3x^3 + 7x^2 + 20x + 60$ r 182

$P(3) = (3)^5 - 2(3)^3 - (3)^2 + 2 = \boxed{182}$