M3

# 6-1 Tangent Lines

Key

## Vocabulary

### Review

1. Cross out the word that does NOT apply to a *circle*.

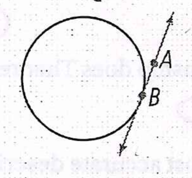
arc circumference diameter ~~equilateral~~ radius

2. Circle the word for a segment with one endpoint at the center of a *circle* and the other endpoint on the *circle*.

arc circumference diameter perimeter radius

### Vocabulary Builder

tangent




**tangent** (noun, adjective) TAN **junt**

**Definition:** A **tangent** to a circle is a line, ray, or segment in the plane of the circle that intersects the circle in exactly one point.

**Other Word Form:** tangency (noun)

**Examples:** In the diagram, $\overleftrightarrow{AB}$ is **tangent** to the circle at *B*. *B* is the *point of tangency*. $\overrightarrow{BA}$ is a **tangent** ray. $\overline{BA}$ is a **tangent** segment.

**Other Usage:** In a right triangle, the **tangent** is the ratio of the side opposite an acute angle to the side adjacent to the angle.

line

Ray

segment

### Use Your Vocabulary

3. Complete each statement with *always*, *sometimes*, or *never*.

A diameter is _?_ a *tangent*. never

A *tangent* and a circle _?_ have exactly one point in common. always

A radius can _?_ be drawn to the point of tangency. always

A *tangent* _?_ passes through the center of a circle. never

A *tangent* is _?_ a ray. sometimes

forms right angle (90°)

## take note — Theorems 12-1, 12-2, and 12-3

⊥ perpendicular

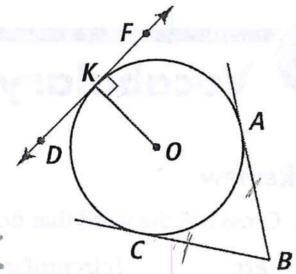
⊙ circle

**Theorem 12-1** If a line is tangent to a circle, then the line is perpendicular to the radius at the point of tangency.

**Theorem 12-2** If a line in the plane of a circle is perpendicular to a radius at its endpoint on the circle, then the line is tangent to the circle.

ABC **Theorem 12-3** If two tangent segments to a circle share a common endpoint outside the circle, then the two segments are congruent.

**Use the diagram at the right for Exercises 4–6. Complete each statement.**




4. **Theorem 12-1** If $\overleftrightarrow{DF}$ is tangent to $\odot O$ at $K$, then DF $\perp$ OK.

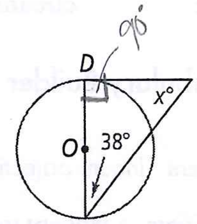
5. **Theorem 12-2** If $\overleftrightarrow{DF} \perp \overline{OK}$, then DF is tangent to $\odot O$.

6. **Theorem 12-3** If $\overline{BA}$ and $\overline{BC}$ are tangent to $\odot O$, then BA $\cong$ BC.

congruent

## Problem 1 Finding Angle Measures

**Got It?** $\overline{ED}$ **is tangent to** $\odot O$**. What is the value of** $x$**?**




7. Circle the word that best describes $\overline{OD}$.

   diameter — radius — tangent

8. What relationship does Theorem 12-1 support? Circle your answer.

   $\overline{OD} \perp \overline{ED}$ — $\overline{OD} \parallel \overline{ED}$ — $\overline{OD} \cong \overline{ED}$

9. Circle the most accurate description of the triangle.

   acute — isosceles — obtuse — right

10. Circle the theorem that you will use to solve for $x$.

    Theorem 12-1 — Triangle Angle-Sum Theorem

    All angles have a sum of 180

11. Complete the model below.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Relate | sum of angle measures in a triangle | is | 38 | plus | measure of ∠D | plus | measure of ∠E |
| Write | 180 | = | 38 | + | 90 | + | X |

12. Solve for $x$.

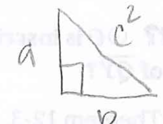
    180 = 38 + 90 + X

    180 = 128 + X

    52 = X

13. The value of $x$ is 52

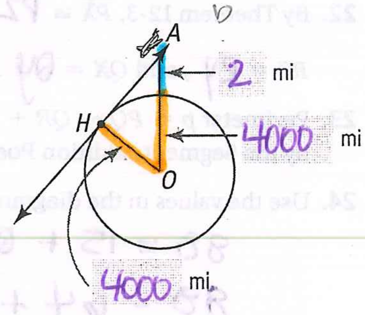
$a^2+b^2=c^2$




## Problem 2 Finding Distance

**Got It?** **What is the distance to the horizon that a person can see on a clear day from an airplane 2 mi above Earth? Earth's radius is about 4000 mi.**

**14.** The diagram at the right shows the airplane at point $A$ and the horizon at point $H$. Use the information in the problem to label the distances.



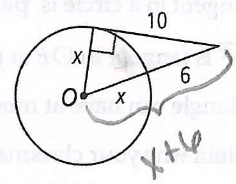

**15.** Use the justifications at the right to find the distance.

| | |
|---|---|
| $OH \perp \overline{AH}$ | Theorem 12-1 |
| $OH^2 + AH^2 = OA^2$ | Pythagorean Theorem |
| $4000^2 + AH^2 = 4002^2$ | Substitute. |
| $16{,}000{,}000 + AH^2 = 16{,}016{,}004$ | Use a calculator. |
| $AH^2 = 16{,}004$ | Subtract from each side. |
| $AH = \sqrt{16{,}004}$ | Take the positive square root. |
| $AH \approx 126.50691$ 68 | Use a calculator. |

**16.** A person can see about 127 miles to the horizon from an airplane 2 mi above Earth.

## Problem 3 Finding a Radius

**Got It?** **What is the radius of $\odot O$?**




**17.** Write an algebraic or numerical expression for each side of the triangle.

$x$ $\qquad$ $10$ $\qquad$ $x+6$

**18.** Circle the longest side of the triangle. Underline the side that is opposite the right angle.

10 $\qquad$ $x$ $\qquad$ $\underline{x + 6}$

**19.** Use the Pythagorean Theorem to complete the equation.

$x^2 + 10^2 = (x+6)^2$ $\qquad$ $a^2+b^2=c^2$ ↑ hypotenuse

**20.** Solve the equation for $x$.

$x^2+10^2=(x+6)^2$

$x^2+100 = x^2+12x+36$

$100 = 12x+36$

$64 = 12x$

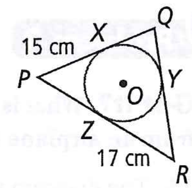
$5\tfrac{1}{3} = x$

**21.** The radius is $5\tfrac{1}{3}$

or $x=5.3$

## Problem 5 Circles Inscribed in Polygons

**Got It?** $\odot O$ is inscribed in $\triangle PQR$, which has a perimeter of 88 cm. What is the length of $\overline{QY}$?




**22.** By Theorem 12-3, $\overline{PX} \cong$ PZ, $\overline{RZ} \cong$ RY, and $\overline{QX} \cong$ QY, so $PX =$ PZ, $RZ =$ RY, and $QX =$ QY.

**23.** Perimeter $p = PQ + QR + RP$, so $p = PX +$ XQ $+ QY +$ YR $+ RZ +$ ZP by the Segment Addition Postulate.

**24.** Use the values in the diagram and your answer to Exercise 23 to solve for $QY$.

$88 = 15 + QX + Qy + 17 + 17 + 15$

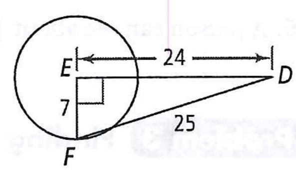
$88 = 64 + 2Qy$

$12 = Qy$

Theorem says $\overline{QX} \cong QY$

## Lesson Check • Do you UNDERSTAND?

**Error Analysis** A classmate insists that $\overline{DF}$ is a tangent to $\odot E$. Explain how to show that your classmate is wrong.




**Underline the correct word or number to complete the sentence.**

**25.** A tangent to a circle is parallel / perpendicular to a radius.

**26.** If $\overline{DF}$ is tangent to $\odot E$ at point $F$, then $m\angle EFD$ must be 30 / 90 / 180 .

**27.** A triangle can have at most 1 right angle(s).

**28.** Explain why your classmate is wrong.

A Δ cannot have two right angles, so the segment is not a tangent

## Math Success

**Check off the vocabulary words that you understand.**

☐ circle ☐ tangent to a circle ☐ point of tangency

**Rate how well you can *use tangents to find missing lengths.***

Need to review 0 2 4 6 8 10 Now I get it!

Key

6-2 Chords and Arcs

## Vocabulary

### Review

**Circle the *converse* of each statement.**

1. **Statement:** If I am happy, then I sing.

If I sing, then I am happy.

If I am not happy, then I do not sing.

If I do not sing, then I am not happy.

2. **Statement:** If parallel lines are cut by a transversal, then alternate interior angles are congruent.

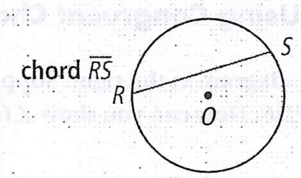
| | | |
|---|---|---|
| If lines cut by a transversal are not parallel, then alternate interior angles are not congruent. | If lines cut by a transversal form alternate interior angles that are not congruent, then the lines are not parallel. | If lines cut by a transversal form alternate interior angles that are congruent, then the lines are parallel. |

### Vocabulary Builder

chord (noun) kawrd

**Definition:** A **chord** is a segment whose endpoints are on a circle.

**Related Word:** arc




### Use Your Vocabulary

3. Complete each statement with *always, sometimes,* or *never.*

A *chord* is _?_ a diameter. sometimes

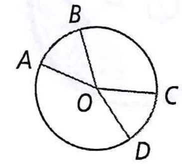
A diameter is _?_ a *chord.* always

A radius is _?_ a *chord.* never

A *chord* _?_ has a related arc. always

An arc is _?_ a semicircle. Sometimes

## take note — Theorems 12-4, 12-5, 12-6 and Their Converses

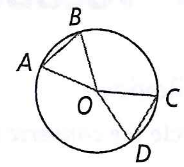



**Theorem 12-4** Within a circle or in congruent circles, congruent central angles have congruent arcs.

4. If $\angle AOB \cong$ $\angle COD$, then $\overparen{AB} \cong \overparen{CD}$.

**Converse** Within a circle or in congruent circles, congruent arcs have congruent central angles.

5. If $\overparen{AB} \cong \overparen{CD}$, then $\angle AOB \cong$ $\angle COD$

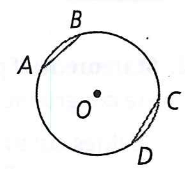



**Theorem 12-5** Within a circle or in congruent circles, congruent central angles have congruent chords.

6. If $\angle AOB \cong \angle COD$, then $\overline{AB} \cong$ $\overline{CD}$.

**Converse** Within a circle or in congruent circles, congruent chords have congruent central angles.

7. If $\overline{AB} \cong \overline{CD}$, then $\angle AOB$ $\cong \angle COD$.




**Theorem 12-6** Within a circle or in congruent circles, congruent chords have congruent arcs.

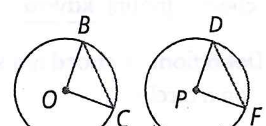
8. If $\overline{AB} \cong$ $\overline{CD}$, then $\overparen{AB} \cong \overparen{CD}$.

**Converse** Within a circle or in congruent circles, congruent arcs have congruent chords.

9. If $\overparen{AB} \cong$ $\overparen{CD}$, then $\overline{AB} \cong \overline{CD}$.

## Problem 1 Using Congruent Chords

**Got It?** Use the diagram at the right. Suppose you are given $\odot O \cong \odot P$ and $\angle OBC \cong \angle PDF$. How can you show $\angle O \cong \angle P$? From this, what else can you conclude?




10. Complete the flow chart below to explain your conclusions.

$\odot O \cong \odot P$ → $\overline{OB} \cong \overline{OC} \cong \overline{PD} \cong \overline{PF}$

Given

$\cong$ circles have $\cong$ radii.

$\angle B \cong \angle C$ and $\angle D \cong \angle F$

Isosceles △ Theorem

$\angle B \cong \angle D$

Given

$\angle C \cong \angle F$

Transitive Property

$\triangle BCO \cong \triangle DFP$

AAS

$\angle O \cong \angle P$

Corresponding parts of $\cong$ △'s are $\cong$.

$\overparen{BC} \cong \overparen{DF}$

Theorem 12-4

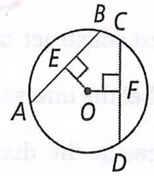
$\overline{BC} \cong \overline{DF}$

Theorem 12-5

## Take note: Theorem 12-7 and Its Converse, Theorems 12-8, 12-9, 12-10

**Theorem 12-7** Within a circle or in congruent circles, chords equidistant from the center or centers are congruent.

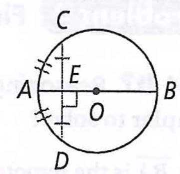
**Converse** Within a circle or in congruent circles, congruent chords are equidistant from the center (or centers).




**11.** If $OE = OF$, then $\overline{AB} \cong$ $\overline{CD}$

**12.** If $\overline{AB} \cong$ $\overline{CD}$, then $OE =$ OF.

**Theorem 12-8** In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.

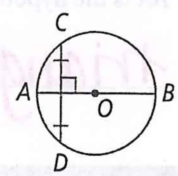



**13.** If $\overline{AB}$ is a diameter and $\overline{AB} \perp \overline{CD}$, then $\overline{CE} \cong$ $\overline{ED}$ and $\overset{\frown}{CA} \cong$ $\overset{\frown}{AD}$.

**Theorem 12-9** In a circle, if a diameter bisects a chord (that is not a diameter), then it is perpendicular to the chord.

**14.** If $\overline{AB}$ is a diameter and $\overline{CE} \cong \overline{ED}$, then $\overline{AB} \perp$ $\overline{CD}$.

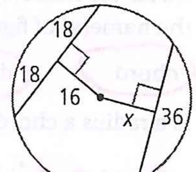
**Theorem 12-10** In a circle, the perpendicular bisector of a chord contains the center of the circle.




**15.** If $\overline{AB}$ is the perpendicular bisector of chord $\overline{CD}$, then $\overline{AB}$ contains the center of $\odot O$.

## Problem 2 Finding the Length of a Chord

**Got It?** What is the value of $x$? Justify your answer.




**16.** What is the measure of each chord? Explain.

The length of one chord is 36 & the other chord has two segments w/ lengths 18, so, by the segment addition postulates, it also has a length of 36

**17.** Circle the reason why the chords are congruent.

| | |
|---|---|
| Chords that have equal measures are congruent. | Chords that are equidistant from the center of a circle are congruent. |

**18.** Circle the theorem that you will use to find the value of $x$.

Theorem 12-5    Theorem 12-7    Converse of Theorem 12-7

Theorem 12-8    Theorem 12-10

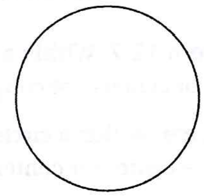
**19.** Circle the distances from the center of a circle to the chords.

16    18    36    $x$

**20.** The value of $x$ is 16.

## Problem 3 Using Diameters and Chords

**Got It?** The diagram shows the tracing of a quarter. What is its radius?



**Underline the correct word to complete each sentence. Then do each step.**

21. First draw two chords / tangents .

22. Next construct one / two perpendicular bisector(s).

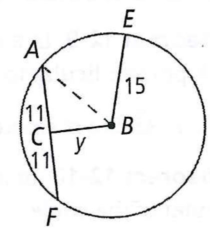
23. Label the intersection $C$. It is the circle's center / chord .

24. Measure the diameter / radius .

25. The radius is about 12 mm.

## Problem 4 Finding Measures in a Circle

**Got It? Reasoning** In finding $y$, how does the auxiliary $\overline{BA}$ make the problem simpler to solve?




26. $\overline{BA}$ is the hypotenuse of a right _?_, so you can use the _?_ Theorem to solve for $y$.

triangle Pythagorean

## Lesson Check • Do you UNDERSTAND?

**Vocabulary** Is a radius a chord? Is a diameter a chord? Explain your answers.

27. Circle the name(s) of figure(s) that have two endpoints on a circle. Underline the name(s) of figure(s) that have one endpoint on a circle.

chord diameter <u>radius</u> ray segment

28. Is a radius a chord? Is a diameter a chord? Explain.

A radius has only one endpoint on a circle so it's not a chord. A diameter has two endpoints on a circle, so it's not a chord.

## Math Success

**Check off the vocabulary words that you understand.**

☐ circle ☐ chord ☐ radius ☐ diameter

**Rate how well you can *use chords to find measures*.**

Need to review 0 2 4 6 8 10 Now I get it!

# 6-3 Inscribed Angles

Key

## Vocabulary

### Review

**Write *noun* or *verb* to identify how *intercept* is used.**

1. Defense tries to *intercept* a touchdown pass. verb

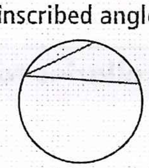
2. The *y-intercept* of a line is the *y*-value at $x = 0$. noun

3. Cryptographers *intercept* and decipher code messages. verb

4. The *x-intercept* of a line is the *x*-value at $y = 0$. noun

### Vocabulary Builder

inscribed angle



**inscribed** (adjective) **in SKRYBD**

**Related Word:** circumscribed

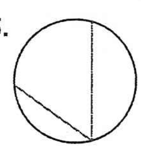
**Definition:** *Inscribed* means written, marked, or engraved on. *Circumscribed* means encircled, confined, or limited.

**Math Usage:** An **inscribed** angle is formed by two chords with a vertex on the circle.

### Use Your Vocabulary

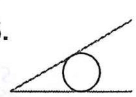
**Write *circumscribed* or *inscribed* to describe each angle.**

5.



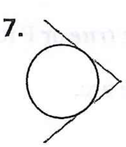
inscribed

6.



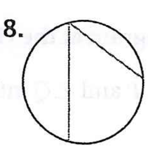
circumscribed

7.



circumscribed

8.



inscribed

**Underline the correct word to complete each sentence.**

9. $\angle ABC$ with points $A$, $B$, and $C$ on a circle is a(n) circumscribed / inscribed angle.

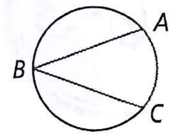
10. An intercepted arc is between the sides of a(n) circumscribed / inscribed angle.

take note

## Theorem 12-11 Inscribed Angle Theorem

The measure of an inscribed angle is half the measure of its intercepted arc.

$m\angle B = \frac{1}{2} m\widehat{AC}$




**11.** Suppose $m\widehat{AC} = 90$.

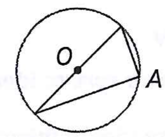
$m\angle B =$ $\frac{1}{2}$ $\cdot\ m\widehat{AC} =$ 45

**12.** Suppose $m\angle B = 60$.

$m\widehat{AC} =$ 2 $\cdot\ m\angle B =$ 120

## Problem 1 Using the Inscribed Angle Theorem

**Got It?** In $\odot O$, what is $m\angle A$?




**13.** Complete the reasoning model below.

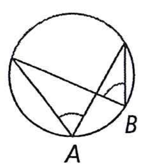
| Think | Write |
|---|---|
| I know the sides of $\angle A$ are chords and the vertex is on $\odot O$. | $\angle A$ is an inscribed angle. |
| I can use the Inscribed Angle Theorem. | $m\angle A = \frac{1}{2}$(measure of the blue arc)<br>$= \frac{1}{2}($180$)$<br>$=$ 90 |

take note

## Corollaries to Theorem 12-11 Inscribed Angle Theorem

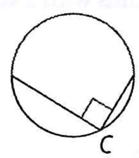
**Corollary 1**

Two inscribed angles that intercept the same arc are congruent.



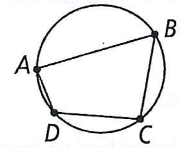

**Corollary 2**

An angle inscribed in a semicircle is a right angle.

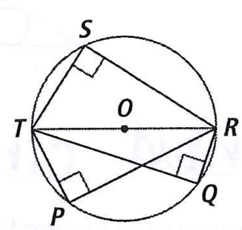



**Corollary 3**

The opposite angles of a quadrilateral inscribed in a circle are supplementary.




**Use the diagram at the right. Write T for *true* or F for *false*.**




T **14.** $\angle P$ and $\angle Q$ intercept the same arc.

F **15.** $\angle SRP$ and $\angle Q$ intercept the same arc.

T **16.** $\widehat{TSR}$ is a semicircle.

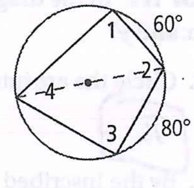
F **17.** $\angle PTS$ and $\angle SRQ$ are opposite angles.

T **18.** $\angle PTS$ and $\angle SRP$ are supplementary angles.

↳ add to be 180°

## Problem 2 Using Corollaries to Find Angle Measures

**Got It?** In the diagram at the right, what is the measure of each numbered angle?




**19.** Use the justifications at the right to complete each statement.

| | |
|---|---|
| $m\angle 4 = \frac{1}{2}($ 60 + 80 $)$ | Inscribed Angle Theorem |
| $m\angle 4 = \frac{1}{2}($ 140 $)$ | Add within parentheses. |
| $m\angle 4 =$ 70 | Simplify. |

**20.** Circle the corollary you can use to find $m\angle 2$.

| | |
|---|---|
| An angle inscribed in a semicircle is a right angle. | The opposite angles of a quadrilateral inscribed in a circle are supplementary. |

**21.** Now solve for $m\angle 2$.

$m\angle 4 + \angle 2 = 180$
$70 + m\angle 2 = 180$
$m\angle 2 = 110$

**22.** Underline the correct word to complete the sentence.

The dashed line is a diameter / radius .

**23.** Circle the corollary you can use to find $m\angle 1$ and $m\angle 3$.

| | |
|---|---|
| An angle inscribed in a semicircle is a right angle. | The opposite angles of a quadrilateral inscribed in a circle are supplementary. |

**Use your answer to Exercise 23 to find the angle measures.**

**24.** $m\angle 1 =$ 90 **25.** $m\angle 3 =$ 90

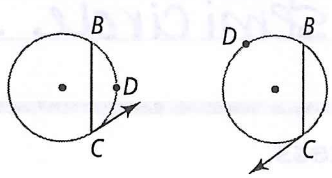
**26.** So, $m\angle 1 =$ 90, $m\angle 2 =$ 110, $m\angle 3 =$ 90, and $m\angle 4 =$ 70.

take note

### Theorem 12-12

The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc.

$m\angle C = \frac{1}{2} m\widehat{BDC}$




**27.** Suppose $m\angle C = 50$.

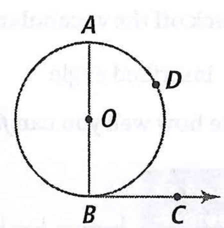
$m\widehat{BDC} =$ 2 $\cdot\, m\angle C =$ 100

**28.** Suppose $m\widehat{BDC} = 80$.

$m\angle C =$ 1/2 $\cdot\, m\widehat{BDC} =$ 40

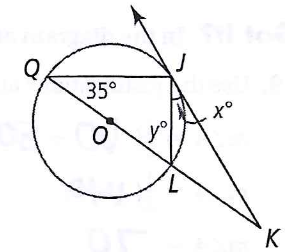
**29.** In the diagram at the right, $\overrightarrow{BC}$ is tangent to $\odot O$ at $B$.

$m\widehat{ADB} =$ 180 $m\angle ABC =$ 90



## Problem 3 Finding Arc Measure

**Got It?** In the diagram at the right, $\overline{KJ}$ is tangent to $\odot O$. What are the values of $x$ and $y$?




**30.** Circle the arc intercepted by $\angle JQL$. Underline the arc intercepted by $\angle KJL$.

$\overset{\frown}{JL}$ $\overset{\frown}{JQ}$ $\overset{\frown}{QL}$ $\overset{\frown}{QLJ}$

**31.** By the Inscribed Angle Theorem, $m\overset{\frown}{JL} = 2 \cdot 35 = 70$.

**32.** By Theorem 12-12, $x = \frac{1}{2} \cdot m\overset{\frown}{JL} = 35$

**33.** The value of $x$ is 35

**34.** Underline the correct words to complete the sentence.

$\overline{QL}$ is a diameter / radius, so $\angle QJL$ is a(n) acute / right / obtuse angle.

**35.** Use the justifications at the right to complete each statement.

$m\angle QJL + m\angle JLQ + m\angle LQJ = 180$ Triangle Angle-Sum Theorem

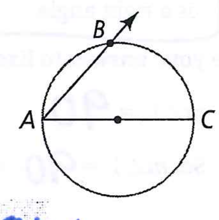
$90 + y + 35 = 180$ Substitute.

$y + 125 = 180$ Simplify.

$y = 55$ Subtract from each side.

## Lesson Check • Do you UNDERSTAND?

**Error Analysis** A classmate says that $m\angle A = 90$. What is your classmate's error?




**36.** Is diameter $\overline{AC}$ a side of $\angle A$? Yes / No

**37.** Is $\angle A$ inscribed in a semicircle? Yes / No

**38.** What is your classmate's error? Explain.

The error is in thinking that an angle w/ diameter as a side intercepts a semicircle. $m\angle A \neq \frac{1}{2} \cdot 180$

## Math Success

Check off the vocabulary words that you understand.

☐ inscribed angle ☐ intercepted arc

Rate how well you can *find the measure of inscribed angles.*

Need to review 0 2 4 6 8 10 Now I get it!

Key

# 6-4 Angle Measures and Segment Lengths

## Vocabulary

### Review

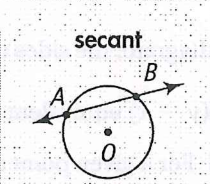
1. Underline the correct word(s) to complete the sentence.

   The student went off on a *tangent* when he did / <u>did not</u> stick to the subject.

2. A *tangent* to a circle intersects the circle at exactly _?_ point(s). one

3. From a point outside a circle, there are _?_ *tangent(s)* to the circle. two

### Vocabulary Builder




**secant** (noun) SEEK unt

**Related Word:** tangent (noun)

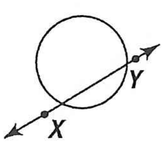
**Definition:** A **secant** is a line that intersects a circle at two points.

**Source:** The word **secant** comes from the Latin verb *secare*, which means "to cut."

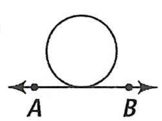
**Examples:** In the diagram at the right, $\overleftrightarrow{AB}$ is a **secant**, $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are **secant** rays, and $\overline{AB}$ is a **secant** segment.

### Use Your Vocabulary

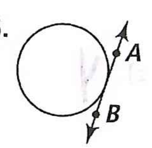
Write *secant* or *tangent* to identify each line.

4. 

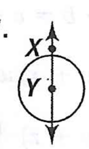

secant

5. 


tangent

6. 


tangent

7. 


secant

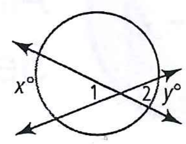
8. Is a chord a *secant*? Explain.

   No; a secant contains points outside a circle, while a chord does not.

## take note — Theorems 12-13, 12-14, and 12-15

**Theorem 12-13** The measure of an angle formed by two lines that intersect inside a circle is half the sum of the measures of the intercepted arcs.

$m\angle 1 = \frac{1}{2}(x + y)$

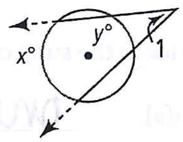


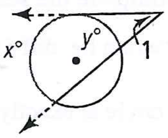

**9.** In the diagram at the right, does $m\angle 2 = \frac{1}{2}(x + y)$? Explain.

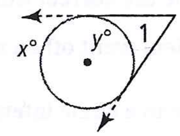
Yes ∠1 & ∠2 are vertical angles so m∠1 = m∠2 & m∠2 = 1/2(x+y) by the Trans. Prop. of Equality

**Theorem 12-14** The measure of an angle formed by two lines that intersect outside a circle is half the difference of the measures of the intercepted arcs.

$m\angle 1 = \frac{1}{2}(x - y)$




**10.** In the first diagam, the sides of the angle are a secant and a __?__. secant

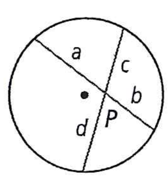
**11.** In the second diagram, the sides of the angle are a secant and a __?__. tangent

**12.** In the third diagram, the sides of the angle are a tangent and a __?__. tangent

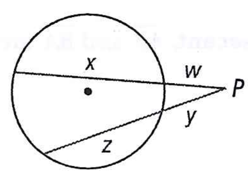
**13.** Is $m\angle 1 = \frac{1}{2}(y - x)$ equivalent to $m\angle 1 = \frac{1}{2}(x - y)$? Yes / No

**Theorem 12-15** For a given point and circle, the product of the lengths of the two segments from the point to the circle is constant along any line through the point and the circle.

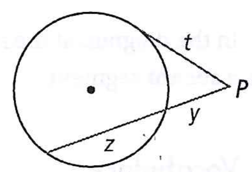
**I.**




**II.**




**III.**




**Complete each case of Theorem 12-15.**

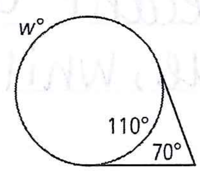
**14. Case I** $a \cdot b = c \cdot$ d

**15. Case II** $(w + x)\,w = ($ y $+ z)$ y

**16. Case III** $(y + z)$ y $= t^2$

## Problem 1 Finding Angle Measures

**Got It?** What is the value of $w$?




**17.** Use Theorem 12-14 to complete the equation.

70 $= \frac{1}{2}(w -$ 110 $)$

**18.** Now solve the equation.

$70 = \frac{1}{2}(w - 110)$

$140 = w - 110$

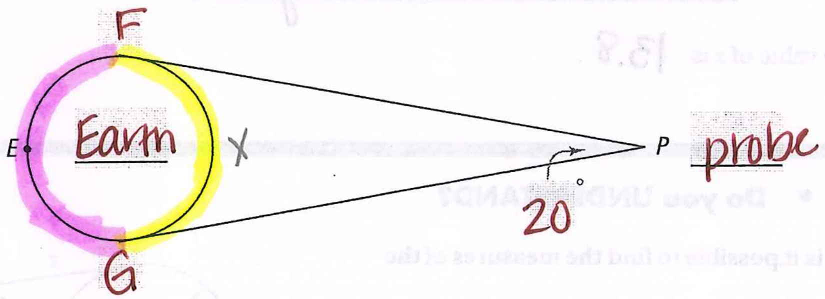
$250 = w$

**19.** The value of $w$ is 250.

## Problem 2 Finding an Arc Measure

**Got It?** A departing space probe sends back a picture of Earth as it crosses Earth's equator. The angle formed by the two tangents to the equator is 20°. What arc of the equator is visible to the space probe?

**20.** Use 20, *F*, *G*, and the words *Earth* and *probe* to complete the diagram below.




**21.** Complete the flow chart below.

Let $m\overset{\frown}{FG} = x$. Then $m\overset{\frown}{FEG} = 360 - x$.
The sum of the arc measures is 360°.

↓

$m\angle FPG = \frac{1}{2}(m\overset{\frown}{FEG} - m\overset{\frown}{FG})$
Theorem 12-14

↓

$20 = \frac{1}{2}(360 - x - x)$
Substitute.

→

$20 = \frac{1}{2}(360 - 2x)$
Simplify.

↓

$20 = 180 - x$
Use the Distributive Property.

←

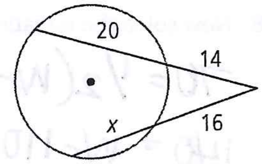
$-160 = -1x$
Subtract 180 from each side.

↓

$160 = x$
Divide each side by –1.

**22.** A 160° arc of the equator is visible to the space probe.

## Problem 3 Finding Segment Lengths




**Got It?** **What is the value of the variable to the nearest tenth?**

**Underline the correct word to complete each sentence.**

**23.** The segments intersect inside / outside the circle.

**24.** Write a justification for each statement.

$(14 + 20)14 = (16 + x)16$ Use Theorem 12-15 (ii)

$476 = 256 + 16x$ Simplify

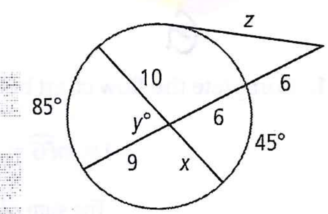
$220 = 16x$ Subtract 256 from each side

$13.75 = x$ divide each side by 16

**25.** To the nearest tenth, the value of $x$ is 13.8 .

## Lesson Check • Do you UNDERSTAND?

**In the diagram at the right, is it possible to find the measures of the unmarked arcs? Explain.**




**26.** You can use intercepted arcs to find the value of $y$. Yes / No

**27.** You can use supplementary angles to find the measures of the angles adjacent to $y°$. Yes / No

**28.** You can find the sum of the unmarked arcs. Yes / No

**29.** Is it possible to find the measure of each unmarked arc? Explain.

No. The sum of the unmarked angle is not enough info. to find the measure of each unmarked arc.

## Math Success

**Check off the vocabulary words that you understand.**

☐ chord ☐ circle ☐ secant ☐ tangent

**Rate how well you can *find the lengths of segments associated with circles.***

Need to review 0 2 4 6 8 10 Now I get it!

Key

# 6-5 Circles in the Coordinate Plane

## Vocabulary

### Review

**Write T for *true* or F for *false*.**

T 1. The *coordinate plane* extends without end and has no thickness.

F 2. Only lines can be graphed in the *coordinate plane*.

T 3. Any polygon can be plotted in the *coordinate plane*.

F 4. (0, 5) and (5, 0) are the same point in the *coordinate plane*.

F 5. The *coordinate plane* is three-dimensional.

T 6. You can find the slope of a line in the *coordinate plane*.

### Vocabulary Builder

**standard form** (noun) STAN durd fawrm

**Main Idea:** The **standard form** of an equation gives information that can help you graph the equation in the coordinate plane.

**Examples:** The **standard form** of an equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$. The **standard form** of a linear equation is $Ax + By = C$. The **standard form** of a quadratic equation is $y = ax^2 + bx + c$.

### Use Your Vocabulary

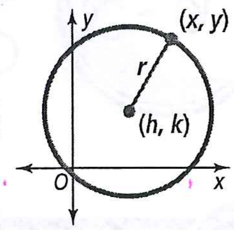
**Draw a line from each equation in Column A to its *standard form* in Column B.**

| Column A | Column B |
|---|---|
| 7. $y = 2x + 3$ | $x + y = 0$ |
| 8. $y = \frac{3}{4}x - 2$ | $2x - y = -3$ |
| 9. $y = -x$ | $3x - 4y = 8$ |
| 10. $0 = 2y - 4x + 3$ | $4x - 2y = 3$ |

take note

## Theorem 12-16 Equation of a Circle

An equation of a circle with center $(h, k)$ and radius $r$ is $(x - h)^2 + (y - k)^2 = r^2$.




**Complete each sentence with *center, circle,* or *radius.***

**11.** Each point on a __?__ is the same distance from the center. circle

**12.** The equation of a circle with center $(-1, 0)$ and __?__ 6 is $(x + 1)^2 + (y - 0)^2 = 6^2$. radius

**13.** Each point on a circle is $r$ units from the __?__. center

**14.** The Distance Formula is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

**15.** How is $d$ in the Distance Formula related to the radius $r$ in the standard equation of a circle?

The radius is the distance d betwen center (h, k) & (x, y) on a circle & is also the hypotenuse of a right △ w/ vertices (h, K), (x, y) & (x-h, y-k).

**16.** How are the Distance Formula and the standard form of the equation of a circle alike?

The standard form of the equation is the distance formula using (x, y) on a circle & the center (h, k). This distane is the radius r of the circle

## Problem 1 Writing the Equation of a Circle

**Got It?** What is the standard equation of the circle with center (3, 5) and radius 6?

**17.** The $x$-coordinate of the center is 3.

**18.** The $y$-coordinate of the center is 5.

**19.** Is the standard equation of a circle $(x - h)^2 + (y - k)^2 = d$ Yes / No

**20.** Identify the values of $h$, $k$, and $r$.

$h =$ 3 $k =$ 5 $r =$ 6

**21.** Write the standard equation of the circle with center (3, 5) and radius 6.

$(x - 3)^2 + (y - 5)^2 = 6^2$

**22.** Simplify the equation in Exercise 21.

$(x - 3)^2 + (y - 5)^2 = 36$

## Problem 2 Using the Center and a Point on a Circle

**Got It?** **What is the standard equation of the circle with center (4, 3) that passes through the point (−1, 1)?**

**23.** Complete the reasoning model below.

| Know | Need | Plan |
|---|---|---|
| $(h, k)$ is $(\boxed{4}, \boxed{3})$. <br> (−1, 1) is a point on the circle. | The radius $\boxed{r}$ <br> The standard equation of the circle | Use the Distance Formula to find $\boxed{r}$. <br> Then substitute for $(h, k)$ and for $\boxed{r}$. |

**24.** Use the Distance Formula to find $r$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Write the Distance Formula.

$r = \sqrt{(4 - -1)^2 + (3 - 1)^2}$ Substitute.

$r = \sqrt{(5)^2 + (2)^2}$ Simplify within parentheses.

$r = \sqrt{(25) + (4)}$ Square each number.

$r = \sqrt{29}$ Add.

**25.** Now write the standard form of the circle with center (4, 3) that passes through the point (−1, 1).

$(x - h)^2 + (y - k)^2 = r^2$ Use the standard form of an equation of a circle.

$(x - 4)^2 + (y - 3)^2 = \sqrt{29}^2$ Substitute.

$(x - 4)^2 + (y - 3)^2 = 29$ Simplify.

## Problem 3 Graphing a Circle Given Its Equation

**Got It?** **Suppose the equation $(x - 7)^2 + (y + 2)^2 = 64$ represents the position and transmission range of a cell tower. What does the center of the circle represent? What does the radius represent?**

**Place a ✓ in the box if the response is correct. Place an ✗ if it is incorrect.**

✓ **26.** The transmission range is the same distance all around the cell tower.

✓ **27.** The center of the circle represents the position of the cell tower.

X **28.** The center of the circle represents the transmission range.

X **29.** The radius of the circle represents the position of the cell tower.

✓ **30.** The radius of the circle represents the transmission range.

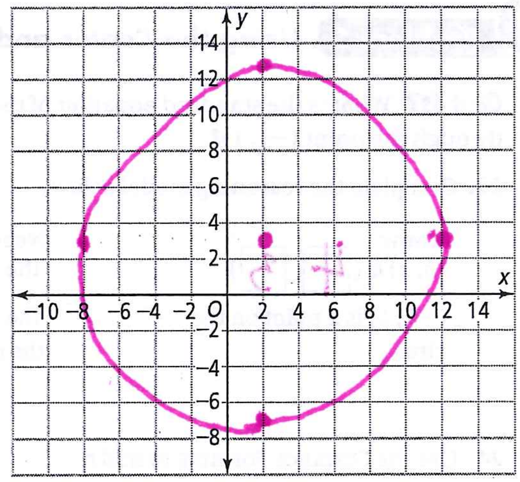
**Got It?** What is the center and radius of the circle with equation $(x - 2)^2 + (y - 3)^2 = 100$? Graph the circle.

**31.** The center of the circle is (2, 3).

**32.** $r^2 =$ 100

**33.** The radius of the circle is 10.

**34.** Graph the circle on the coordinate plane at the right.




## Lesson Check • Do you UNDERSTAND?

**Suppose you know the center of a circle and a point on the circle. How do you determine the equation of the circle?**

**35.** Do you know the value of $h$? Yes / No

**36.** Do you know the value of $k$? Yes / No

**37.** Do you know the value of $r$? Yes / No

**38.** How can you find the missing value?

Use the distance formula to find the radius r of the circle

**39.** Once you know $h$, $k$, and $r$, how do you determine an equation of the circle?

Sustitute the values in the standard equation of a circle.

## Math Success

**Check off the vocabulary words that you understand.**

☐ circle ☐ Distance Formula ☐ standard form

**Rate how well you can *use the standard form of a circle.***

Need to review 0 2 4 6 8 10 Now I get it!