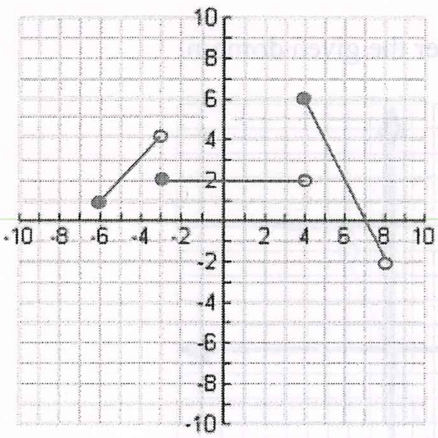
Math III H Intro to Piece-Wise Functions

Chapter 7

Key

The following graph is called a **piecewise function** because the function is defined by two or more different equations applied to different parts of the function's domain.



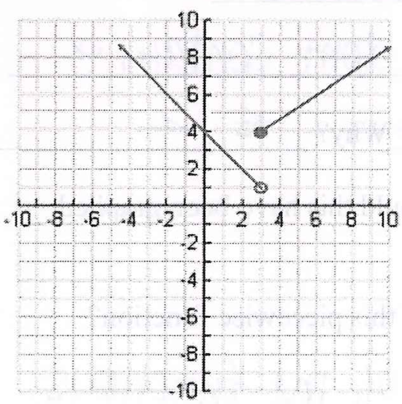
Notice that it appears to be composed of three segments, each a different linear function over a particular domain. Please note a filled circle includes that point, while an open circle does not include that point.

○ Not included >, < ( )

● included ≥ ≤ [ ]

1. What is the domain (x-values) for the first (left) segment? [-6, -3) the range (y-values)? [1, 4)
2. What is the domain for the second (middle) segment? [-3, 4) the range? 2
3. What is the domain for the third (right) segment? [4, 8) the range? [6, -2)

4. How many equations do you think you would have to use to write the rule for the following piecewise function? 3



arrows @ one end

Notice that it appears to be composed of two rays, each a different linear function over a particular domain.

* infinity always has parenthesis *

5. What is the domain for the first (left) ray? (-∞, 3) the range? [1, ∞)
6. What is the domain for the second (right) ray? [3, ∞) the range? [4, ∞)

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Given

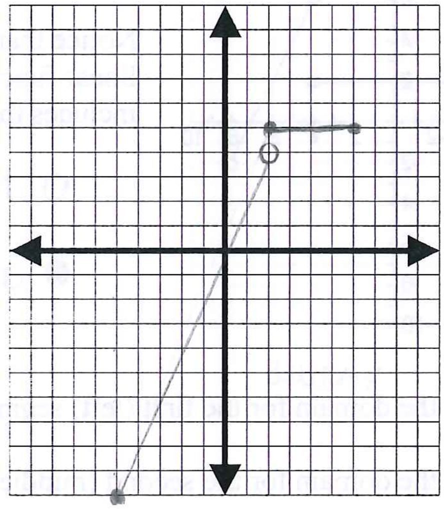
$$f(x) = \begin{cases} 2x & ,-5 \le x < 2 \quad \text{linear} \\ 5 & ,2 \le x \le 6 \quad \text{horizontal} \end{cases}$$

1. Complete the following table of values for the piecewise function over the given domain.

1st function

x	$f(x)$
-5	-10
-3	-6
0	0
1	2
1.7	3.4
1.9	3.8
2	4
2	5
2.2	5
4	5
6	5

2nd function



2. Graph the ordered pairs from your table to hand sketch the graph of the piecewise function.

3. How many pieces does your graph have? 2 Why? 2 equations

4. Are the pieces rays or segments? segments Why? don't contain ∞

5. Are all the endpoints solid dots or open dots or some of each? some of each Why? $\le$, $<$

6. Were all these x values necessary to graph this piecewise function, or could this have been graphed using less points? less points

7. Which x values were "critical" to include in order to sketch the graph of this piecewise function?

-5, 2, 6, these are the constraints of the functions. They tell you where to stop & start.

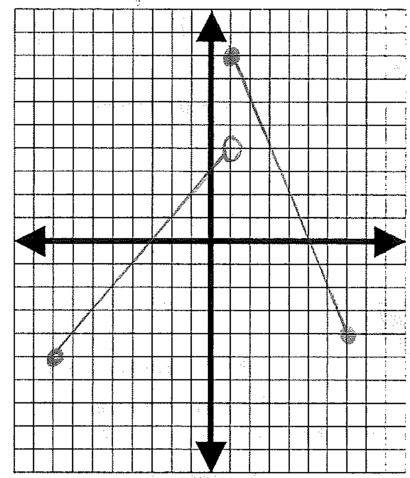
8. $f(x) = \begin{cases} x+3 & ,-8 \le x < 1 \\ 10-2x & ,1 \le x \le 7 \end{cases}$

a. Make a table of values for the piecewise function over the given domain.

x	$f(x)$
-8	-5
1	4
1	8
7	-4

b. Why did you choose the x values you placed into the table? Constraints given

c. Graph the ordered pairs from your table to hand sketch the graph of the piecewise function.



d. How many pieces does your graph have? 2 Why? 2 functions

e. Are the pieces rays or segments? segments Why? don't contain ∞

f. Are all the endpoints filled circles or open circles or some of each? Some of each Why? ≤, <

g. Was it necessary to evaluate both pieces of the function for the x-value 1? yes

Why or why not? In order to create the segment needed.

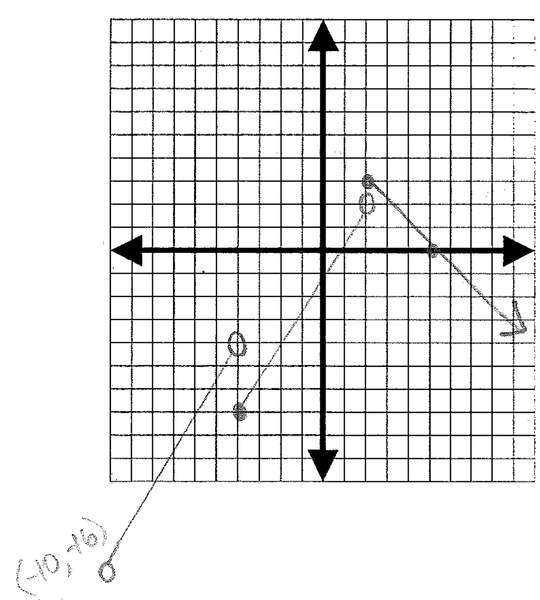
h. Which x values were "critical" to include in order to graph this piecewise function? Explain.

−8, 1, 7 these are the constraints for the function They tell you were to stop & stop the graph

Complete a table of values for the piecewise functions over the given domains.

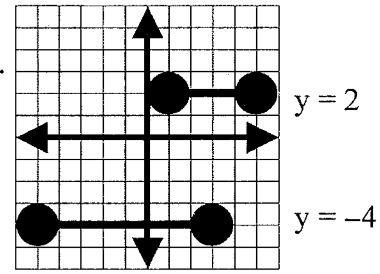
9. $f(x)=\begin{cases} 2x+4 & \text{if } -10<x<-4 \\ \frac{3}{2}x-1 & \phantom{\text{if }} -4\le x<2 \\ -x+5 & \phantom{\text{if }} x\ge 2 \end{cases}$ seg / seg / ray

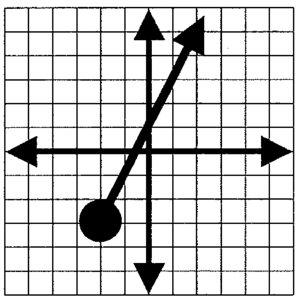
x	$f(x)$	
−10	−16	o
−4	−4	o
−4	−7	•
2	2	o
2	3	•
5	0	→



Writing Equations of Piecewise Defined Functions

The EASIEST equation to write is that of a horizontal line. The equation of a horizontal line is always written like this: y = b. where b is the y-intercept. So the equation for the TOP line at the right would be y = 2 with a restricted domain of $1 \le x \le 5$.
The line at the bottom of the graph is y = –4 with a domain of $-5 \le x \le 3$.





This equation is linear. You can see where the line crosses the y-axis (the **y-intercept** or **b**) and you can easily count the slope of the line (**m**). This will allow you to write the equation in slope intercept form (**y = mx + b**). The graph at the left is y = 2x + 1 with a domain of $x \ge -2$.

closed circle

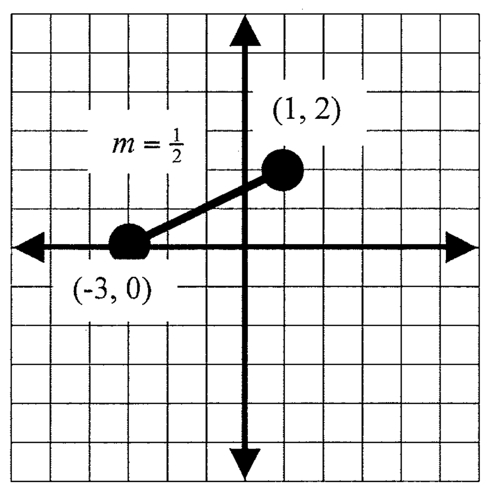
$$y = mx + b$$

$slope = \dfrac{rise}{run}$ *y-intercept*

Sometimes you don't have a y-intercept that is an integer, or the y-intercept cannot be seen on the graph. You always have a y-intercept unless the line is vertical. If this is the case, then you have to use point slope form. This requires you to know the slope of the line and 2 points on the line.

The graph at the right has a y-intercept that you can see, but it is not one that you can readily determine. You never want to guess what the y-intercept is if it is not an integer.
So we will use the point-slope formula to determine the equation of the line.

Point-slope Form: $y - y_1 = m(x - x_1)$



Step 1: Find the slope of the line. You can either….

a. count rise and run $m = \dfrac{rise}{run}$

b. use the two endpoints and slope formula $\dfrac{y_2 - y_1}{x_2 - x_1}$

Step 2: Take ONE of the endpoints and the slope and plug into the point slope form. $m = \frac{1}{2}$ and use (1, 2)

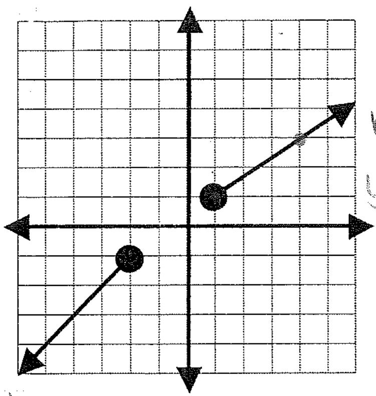
$y - 2 = \frac{1}{2}(x - 1)$
$y - 2 = \frac{1}{2}x - \frac{1}{2}$
$y = \frac{1}{2}x + 1\frac{1}{2}$

$\boxed{y = \frac{1}{2}x + 1\frac{1}{2} \quad -3 \le x \le 1}$

Math III H Intro to Piece-Wise Functions $y=a(x-h)^2+K$ x,y point h,k vertex vertex (0,1)

Write **equations** for the piecewise functions whose graphs are shown below. Include the **domain** for each equation.

1.



(1,1)

$m=\frac{2}{3}$ $y-1=\frac{2}{3}(x-1)$

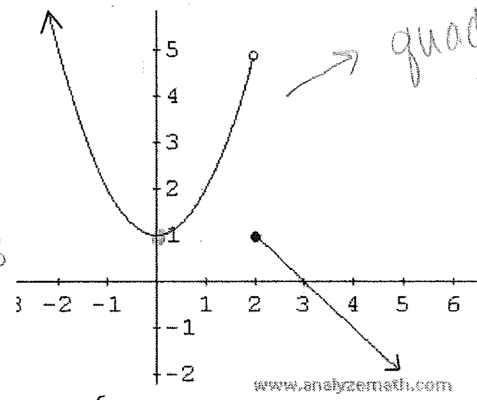
yint $=\frac{1}{3}$ $y-1=\frac{2}{3}x-\frac{2}{3}$

$y=\frac{2}{3}x+\frac{1}{3}$

$y=x+1$ m=1 yint=1

$$f(x)=\begin{cases} x+1 & \text{if } x\le -2 \\ \frac{2}{3}x+\frac{1}{3} & x\ge 1 \end{cases}$$

2.



quadratic x^2

$2=a(1-0)^2+1$

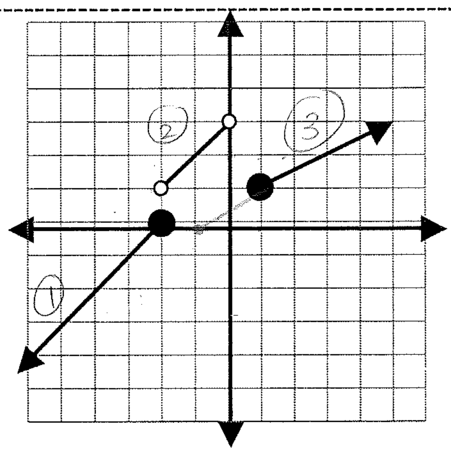
$2=a+1$

$a=1$

$y=1(x-0)^2+1$

$$f(x)=\begin{cases} x^2+1 & \text{if } x<2 \\ -x+3 & x\ge 2 \end{cases}$$

3.

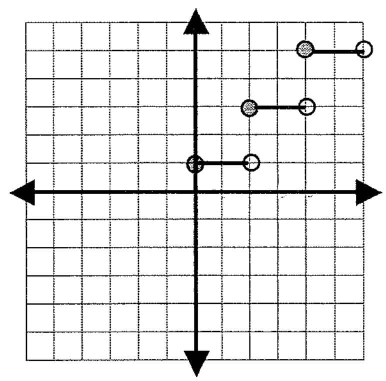


$y-1=\frac{1}{2}(x-1)$

$y=\frac{1}{2}x-0.5$

$$f(x)=\begin{cases} x+2 & x\le -2 \\ x+3 & \text{if } -2<x<0 \\ \frac{1}{2}x+0.5 & x\ge 1 \end{cases}$$

4.



$$f(x)=\begin{cases} 1 & 0\le x<2 \\ 3 & \text{if } 2\le x<4 \\ 5 & 4\le x<6 \end{cases}$$

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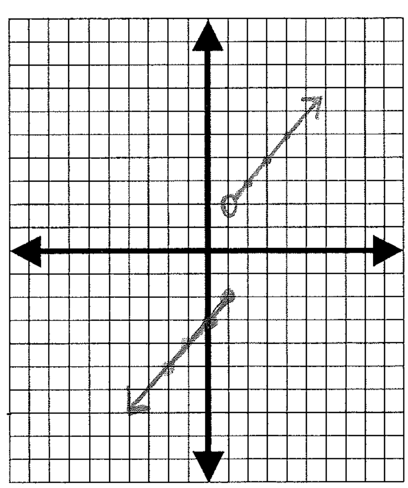
5. $f(x)=\begin{cases} x-3 & x\le 1 \\ x+1 & x>1 \end{cases}$



6. $f(x)=\begin{cases} -x & x<-1 \\ -2 & -1\le x<2 \\ 2x & x\ge 2 \end{cases}$