Name_____ Class _____ Date _____

3-1 Practice

Form G

Exploring Exponential Models

Graph each function.

1. $y = (0.3)^x$
2. $y = 3^x$
3. $y = 2(\frac{1}{5})^x$
4. $y = \frac{1}{2}(3)^x$
5. $s(t) = 2.5^t$
6. $f(x) = \frac{1}{2}(5)^x$

Without graphing, determine whether the function represents exponential growth or exponential decay. Then find the *y*-intercept.

7. $y = 0.99\left(\frac{1}{3}\right)^x$
8. $y = 20(1.75)^x$
9. $y = 185(\frac{5}{4})^x$
10. $f(x) = \frac{2}{3}\left(\frac{1}{2}\right)^x$
11. $f(x) = 0.25(1.05)^x$
12. $y = \frac{1}{5}\left(\frac{6}{5}\right)^x$

13. Suppose you deposit $1500 in a savings account that pays interest at an annual rate of 6%. No money is added or withdrawn from the account.
    a. How much will be in the account after 5 years?
    b. How much will be in the account after 20 years?
    c. How many years will it take for the account to contain $2500?
    d. How many years will it take for the account to contain $4000?

Write an exponential function to model each situation. Find each amount after the specified time.

14. A population of 1,236,000 grows 1.3% per year for 10 years.

15. A population of 752,000 decreases 1.4% per year for 18 years.

16. A new car that sells for $18,000 depreciates 25% each year for 4 years.

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3-1 Practice (continued) *Form G*

Exploring Exponential Models

For each annual rate of change, find the corresponding growth or decay factor.

17. + 45%

18. − 10%

19. − 40%

20. + 200%

21. + 28%

22. + 100%

23. − 5%

24. + 3%

25. In 2009, there were 1570 bears in a wildlife refuge. In 2010, the population had increased to approximately 1884 bears. If this trend continues and the bear population is increasing exponentially, how many bears will there be in 2010?

26. The value of a piece of equipment has a decay factor of 0.80 per year. After 5 years, the equipment is worth $98,304. What was the original value of the equipment?

27. Your friend drops a rubber ball from 4 ft. You notice that its rebound is 32.5 in. on the first bounce and 22 in. on the second bounce.

a. What exponential function would be a good model for the height of the ball?

b. How high will the ball bounce on the fourth bounce?

28. An investment of $75,000 increases at a rate of 12.5% per year. What is the value of the investment after 30 years?

29. A new truck that sells for $29,000 depreciates 12% each year. What is the value of the truck after 7 years?

30. The price of a new home is $350,000. The value of the home appreciates 2% each year. How much will the home be worth in 10 years?

31. The population of an endangered bird is decreasing at a rate of 0.75% per year. There are currently about 200,000 of these birds.

a. What exponential function would be a good model for the population of these endangered birds?

b. How many birds will there be in 100 years?

Name_____ Class _____ Date_____

3-2 Practice

Form G

Properties of Logarithms

Write each expression as a single logarithm.

1. $\log_5 4 + \log_5 3$

2. $\log_6 25 - \log_6 5$

3. $\log_2 4 + \log_2 2 - \log_2 8$

4. $5\log_7 x = 2\log_7 x$

5. $\log_4 60 - \log_4 4 + \log_4 x$

6. $\log 7 - \log 3 + \log 6$

7. $2\log x - 3\log y$

8. $\frac{1}{2}\log r + \frac{1}{3}\log s - \frac{1}{4}\log t$

9. $\log_3 4x + 2\log_3 5y$

10. $5\log 2 - 2\log 2$

11. $\frac{1}{3}\log 3x + \frac{2}{3}\log 3x$

12. $2\log 4 + \log 2 + \log 2$

13. $(\log 3 - \log 4) - \log 2$

14. $5\log x + 3\log x^2$

15. $\log_6 3 - \log_6 6$

16. $\log 2 + \log 4 - \log 7$

17. $\log_3 2x - 5\log_3 y$

18. $\frac{1}{2}(\log_2 x - \log_2 y)$

19. $\frac{1}{2}\log x + \frac{1}{3}\log y - 2\log z$

20. $3(4\log t^2)$

21. $\log_5 y - 4(\log_5 r + 2\log_5 t)$

Expand each logarithm. Simplify if possible.

22. $\log xyz$

23. $\log_2 \frac{x}{yz}$

24. $\log 6x^3 y$

25. $\log 7(3x-2)^2$

26. $\log \sqrt{\frac{2rst}{5w}}$

27. $\log \frac{5x}{4y}$

28. $\log_5 5x^{-5}$

29. $\log \frac{2x^2 y}{3k^3}$

30. $\log_4 (3xyz)^2$

Use the Change of Base Formula to evaluate each expression. Round your answer to the nearest thousandth.

31. $\log_4 32$

32. $\log_3 5$

33. $\log_2 15$

34. $\log_6 17$

35. $\log_6 10$

36. $\log_5 6$

37. $\log_8 1$

38. $\log_9 11$

39. The concentration of hydrogen ions in a batch of homemade ketchup is 10^{-4}. What is the pH level of the ketchup?

Name_____ Class _____ Date_____

3-2 Practice (continued) Form G

Properties of Logarithms

Determine if each statement is *true* or *false*. Justify your answer.

40. $\log 12 = \log 4 + \log 3$

41. $\log \frac{3}{5} = \frac{\log 3}{\log 5}$

42. $\log_6 12 + \log_6 3 = 2$

43. $\frac{1}{2}\log_4 4x = \log_4 2x$

Use the properties of logarithms to evaluate each expression.

44. $\log_2 8 + \log_2 4$

45. $\log_2 160 - \log_2 5$

46. $\log_6 27 + \log_6 8$

47. $\log_7 14 - \log_7 2$

48. $\log_4 64 + 2\log_4 2$

49. $\frac{1}{4}\log_3 162 - \log_3 \sqrt[4]{2}$

State the property or properties used to rewrite each expression.

50. $\log 6 - \log 3 = \log 2$

51. $6 \log 2 = \log 64$

52. $\log 3x = \log 3 + \log x$

53. $\frac{1}{3}\log_2 x = \log_2 \sqrt[3]{x}$

54. $\frac{2}{3}\log 7 = \log \sqrt[3]{49}$

55. $\log_4 20 - 3\log_4 x = \log_4 \frac{20}{x^3}$

The formula for loudness in decibels (dB) is $L = 10 \log \frac{I}{I_0}$, where I is the intensity of a sound in watts per square meter (W/m^2) and I_0 is 10^{-12} W/m^2, the intensity of a barely audible sound.

56. A sound has an intensity of 5.92×10^{25} W/m^2. What is the loudness of the sound in decibels? Use $I_0 = 10^{-12}$ W/m^2.

57. Suppose you decrease the intensity of a sound by 45%. By how many decibels would the loudness be decreased?

58. Writing Explain why $\left(\frac{9}{4}\right) \neq \frac{\log 9}{\log 4}$.

Name_____ Class _____ Date _____

3-3 Practice

Form G

Exponential and Logarithmic Equations

Solve each equation.

1. $8^{2x} = 32$
2. $7^{n} = 343$
3. $9^{2x} = 27$
4. $25^{2n+1} = 625$
5. $36^{-2x+1} = 216$
6. $64^{x} = 4096$

Solve each equation. Round answers to the nearest hundredth.

7. $5^{2x} = 20$
8. $8^{n+1} = 3$
9. $4^{n-2} = 3$
10. $4^{3n} = 5$
11. $15^{2n-3} = 245$
12. $4^{x} - 5 = 12$

Solve by graphing. Round to the nearest hundredth.

13. $2^{n+5} = 120$
14. $5^{n+1} = 175$
15. $8^{x} = 58$
16. $10^{n} = 3$
17. $10^{3y} = 5$
18. $10^{k-2} = 20$
19. $5^{x} = 4$
20. $2^{4x} = 8$
21. $3^{x+5} = 15$

Use a table to solve each equation. Round to the nearest hundredth.

22. $8^{2n} = 3$
23. $12^{2n-1} = 64$
24. $12^{n-2} = 84$
25. $10^{x} = 182$
26. $8^{n} = 12$
27. $10^{2x} = 9$
28. $5^{n+1} = 3$
29. $10^{n-2} = 0.3$
30. $3^{3n} = 50$

31. The equation $y = 281(1.01)^{x}$ is a model for the population of the United States y, in millions of people, x years after the year 2000. Estimate when the United States population will reach 400 million people.

Solve each equation. Check your answers.

32. $\log x = 2$
33. $\log 4x = -1$
34. $\log 3x = 2$
35. $\log 4x = 2$
36. $4 \log x = 4$
37. $8 \log x = 16$
38. $2 \log x = 2$
39. $\log (2x + 5) = 3$
40. $\log (3x - 2)\ 5=3$
41. $\log (x - 25) = 2$
42. $2 \log (2x + 5) = 4$
43. $3 \log (1 - 2x) = 6$

Name_____ Class _____ Date _____

3-3 Practice (continued) *Form G*

Exponential and Logarithmic Equations

Solve each equation.

44. $\log x - \log 4 = 3$

45. $\log x - \log 4 = -2$

46. $2\log x - \log 4 = 2$

47. $\log 3x - \log 5 = 1$

48. $2\log x - \log 3 = 1$

49. $\log 8 - \log 2x = -1$

50. $2\log 3x - \log 9 = 1$

51. $2\log x - \log 5 = -2$

52. $\log (x + 21) + \log x = 2$

53. The function $y = 1000(1.005)^x$ models the value of \$1000 deposited at an interest rate of 6% per year (0.005 per month) x months after the money is deposited.

a. Use a graph (on your graphing calculator) to predict how many months it will be until the account is worth \$1100.

b. Predict how many years it will be until the account is worth \$5000.

54. Suppose the population of a country is currently 8,100,000. Studies show this country's population is increasing 2% each year.

a. What exponential function would be a good model for this country's population?

b. Using the equation you found in part (a), how many years will it take for the country's population to reach 9 million? Round your answer to the nearest hundredth.

55. Suppose you deposit \$2500 in a savings account that pays you 5% interest per year.

a. How many years will it take for you to double your money?

b. How many years will it take for your account to reach \$8,000?

Mental Math Solve each equation.

56. $5x = \frac{1}{25}$

57. $4^x = 64$

58. $10^x = 0.0001$

59. $\log 81 = x$

60. $\log_2 \frac{1}{32} = x$

61. $\log 1{,}000{,}000 = x$

Use the properties of exponential and logarithmic functions to solve each system. Check your answers.

62. $\begin{cases} -2^{10-x} + y = 0 \\ y = 8^{x+2} \end{cases}$

63. $\begin{cases} 3^{2x-y} = 1 \\ 4^{x+y} - 8 = 0 \end{cases}$

64. $\begin{cases} \log_2 (x - 2y) = 3 \\ \log_2 (x + y) = \log_2 8 \end{cases}$

Name_____ Class_____ Date_____

3-4 Practice

Form G

Natural Logarithms

Write each expression as a single natural logarithm.

1. $\ln 16 - \ln 8$

2. $3 \ln 3 + \ln 9$

3. $a \ln 4 - \ln b$

4. $\ln z - 3 \ln x$

5. $\frac{1}{2} \ln 9 + \ln 3x$

6. $4 \ln x + 3 \ln y$

7. $\frac{1}{3} \ln 8 + \ln x$

8. $3 \ln a - b \ln 2$

9. $2 \ln 4 - \ln 8$

Solve each equation. Check your answers. Round your answer to the nearest hundredth.

10. $4 \ln x = -2$

11. $2 \ln (3x - 4) = 7$

12. $5 \ln (4x - 6) = -6$

13. $-7 + \ln 2x = 4$

14. $3 - 4 \ln (8x + 1) = 12$

15. $\ln x + \ln 3x = 14$

16. $2 \ln x + \ln x^2 = 3$

17. $\ln x + \ln 4 = 2$

18. $\ln x - \ln 5 = -1$

19. $\ln e^x = 3$

20. $3 \ln e^{2x} = 12$

21. $\ln e^{x+5} = 17$

22. $\ln 3x + \ln 2x = 3$

23. $5 \ln (3x - 2) = 15$

24. $7 \ln (2x + 5) = 8$

25. $\ln (3x + 4) = 5$

26. $\ln \frac{2x}{41} = 2$

27. $\ln (2x - 1)^2 = 4$

Use natural logarithms to solve each equation. Round your answer to the nearest hundredth.

28. $e^x = 15$

29. $4e^x = 10$

30. $e^x 1 - = 50$

31. $4e^{3x21} = 5$

32. $e^{x.4} = 2$

33. $5e^{6x+3} = 0.1$

34. $e^x = 1$

35. $e^{\frac{x}{5}} = 32$

36. $3e^{3x.5} = 49$

37. $7e^{5x+8} = 0.23$

38. $6 - e^{12x} = 5.2$

39. $e^{\frac{x}{5}} = 25$

40. $e^{2x} = 25$

41. $e^{\ln 5x} = 20$

42. $e^{\ln x} = 21$

43. $e^{x+6} + 5 = 1$

3-4 Practice (continued) Form G

Natural Logarithms

The formula $P = 50e^{-\frac{t}{25}}$ gives the power output P, in watts, needed to run a certain satellite for t days. Find how long a satellite with the given power output will operate.

44. 10 W **45.** 12 W **46.** 14 W

The formula for the maximum velocity v of a rocket is $v = -0.0098t + c \ln R$, where c is the exhaust velocity in km/s, t is the f ring time, and R is the mass ratio of the rocket. A rocket must reach 7.7 km/s to attain a stable orbit 300 km above Earth.

47. What is the maximum velocity of a rocket with a mass ratio of 18, an exhaust velocity of 2.2 km/s, and a firing time of 25 s?

48. Can the rocket in Exercise 47 achieve a stable orbit? Explain your answer.

49. What mass ratio would be needed to achieve a stable orbit for a rocket with an exhaust velocity of 2.5 km/s and a f ring time of 29 s?

50. A rocket with an exhaust velocity of 2.4 km/s and a 28 second f ring time can reach a maximum velocity of 7.8 km/s. What is the mass ratio of the rocket?

By measuring the amount of carbon-14 in an object, a paleontologist can determine its approximate age. The amount of carbon-14 in an object is given by $y = ae^{-0.00012t}$, where a is the amount of carbon-14 originally in the object, and t is the age of the object in years.

51. A fossil of a bone contains 32% of its original carbon-14. What is the approximate age of the bone?

52. A fossil of a bone contains 83% of its original carbon-14. What is the approximate age of the bone?

Simplify each expression.

53. $\ln e^4$ **54.** $5 \ln e^5$ **55.** $\frac{\ln e^2}{2}$ **56.** $\ln e^{100}$