6-8 Areas of Parallelograms and Triangles
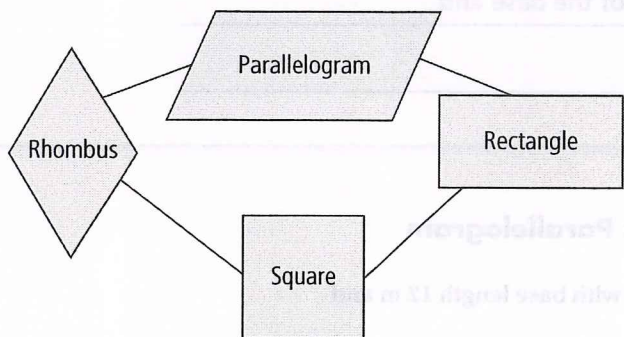
Vocabulary

Review

The diagram below shows the different types of *parallelograms*.

Parallelogram

Rhombus

Rectangle

Square

Underline the correct word to complete each sentence.

1. All *parallelograms* are <u>quadrilaterals</u> / rectangles .

2. All *parallelograms* have opposite sides <u>parallel</u> / perpendicular .

3. Some *parallelograms* are trapezoids / <u>rectangles</u> .

Vocabulary Builder

area (noun) EHR <u>ee uh</u>

Definition: **Area** is the number of square units needed to cover a given surface.
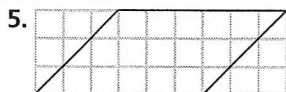
Main Idea: You can find the **area** of a parallelogram or a triangle when you know the length of its base and its height.
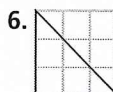
Use Your Vocabulary

Find the area of each figure.

4.

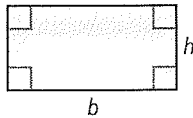18 square units

5.

18 square units

6.

4.5 square units

Theorems 10-1 and 10-2 Area of a Rectangle and a Parallelogram

Theorem 10-1 Area of a Rectangle
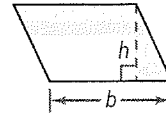The area of a rectangle is the product of its base and height.

$$A = bh$$

h

b

Theorem 10-2 Area of a Parallelogram
The area of a parallelogram is the product of a base and the corresponding height.

$$A = bh$$

h

b

7. Explain how finding the area of a parallelogram and finding the area of a rectangle are alike. **Explanations may vary. Sample:**

For each figure, you find the product of the base and

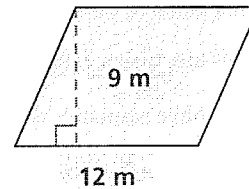its corresponding height.

Problem 1 Finding the Area of a Parallelogram

Got It? What is the area of a parallelogram with base length 12 m and height 9 m?

8. Label the parallelogram at the right.

9. Find the area.

   $A = bh$ Write the formula.

   $= 12(\ 9 \ )$ Substitute.

   $= 108$ Simplify.

9 m

12 m

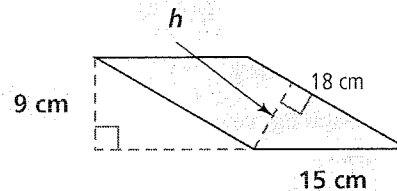10. The area of the parallelogram is 108 m^2.

Problem 2 Finding a Missing Dimension

Got It? A parallelogram has sides 15 cm and 18 cm. The height corresponding to a 15-cm base is 9 cm. What is the height corresponding to an 18-cm base?

11. Label the parallelogram at the right.
    Let h represent the height corresponding to the 18-cm base.

12. Find the area.

   $A = bh$
   $= 15 \cdot 9$
   $= 135$

h

9 cm

18 cm

15 cm

13. The area of the parallelogram is 135 cm^2.

14. Use the area of the parallelogram to find the height corresponding to an 18-cm base.

$A = bh$ Write the formula.

$135 = (\ 18\ )h$ Substitute.

$\dfrac{135}{18} = \dfrac{(\ 18\ )h}{18}$ Divide each side by the length of the base.
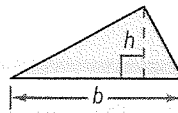
$7.5 = h$ Simplify.

15. The height corresponding to an 18-cm base is 7.5 cm.

---

take note

Theorem 10-3   Area of a Triangle

The area of a triangle is half the product of a base and the corresponding height.

$A = \frac{1}{2}bh$

16. Explain how finding the area of a triangle is different from finding the area of a rectangle. **Explanations may vary. Sample:**

For a triangle, find half the product of the base and height. For a rectangle, find the product of the base and height.

Problem 3 Finding the Area of a Triangle

Got It? What is the area of the triangle?

17. Circle the formula you can use to find the area of the triangle.

$A = bh$ $A \boxed{= \frac{1}{2}bh}$

18. Convert the lengths of the base and the hypotenuse to inches.
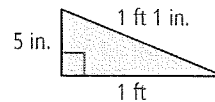
base   hypotenuse

1 ft = 12 in.   1 ft 1 in. = 13 in.

19. Find the area of the triangle.

$A = \frac{1}{2}bh$

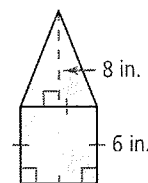$= \frac{1}{2}(12)(5)$

$= \frac{1}{2}(60)$

$= 30$

20. The area of the triangle is 30 in.^{2}.

Problem 4 Finding the Area of an Irregular Figure

Got It? Reasoning Suppose the base lengths of the square and triangle in the figure are doubled to 12 in., but the height of each polygon remains the same. How is the area of the figure affected?

8 in.

6 in.

21. Complete to find the area of each irregular figure.

Area of Original Irregular Figure	Area of New Irregular Figure

$$A = 6(6) + \frac{1}{2}(6)(8)$$

$$= \boxed{36} + 24$$

$$= 60$$

$$A = (2)(6)(6) + \frac{1}{2}(2)(6)(8)$$

$$= (2)(36) + (2)(\boxed{24})$$

$$= (2)(\boxed{36} + \boxed{24}) = (2)(\boxed{60}) = \boxed{120}$$

22. How is the area affected?

The area is doubled.

Lesson Check ⊕ Do you UNDERSTAND?

$\square ABCD$ is divided into two triangles along diagonal $\overline{AC}$. If you know the area of the parallelogram, how do you find the area of $\triangle ABC$?
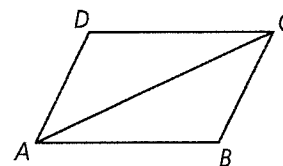
Write T for *true* or F for *false*.

_T_ 23. Since $\overline{AC}$ is a diagonal of $\square ABCD$, $\triangle ABC$ is congruent to $\triangle CDA$.

_F_ 24. The area of $\triangle ABC$ is greater than the area $\triangle CDA$.

_T_ 25. The area of $\triangle ABC$ is half the area of $\square ABCD$.

26. If you know the area of the parallelogram, how do you find the area of $\triangle ABC$?

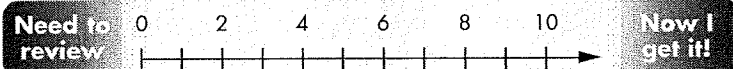Divide the area of the parallelogram by 2.

Math Success

Check off the vocabulary words that you understand.

☐ base of a parallelogram   ☐ height of a parallelogram

☐ base of a triangle   ☐ height of a triangle

Rate how well you can *find the area of parallelograms and triangles.*

Need to review   0   2   4   6   8   10   Now I get it!

Space Figures and Cross Sections

Vocabulary

● Review

Complete each statement with the correct word from the list.

edge	edges	vertex	vertices

1. A(n) __?__ is a segment that is formed by the intersections of two faces.

> edge

2. A(n) __?__ is a point where two or more *edges* intersect.

> vertex

3. A cube has eight __?__ .

> vertices

4. A cube has twelve __?__ .

> edges

● Vocabulary Builder

polyhedron (noun) **pahl ih HEE drun** (plural: polyhedra)

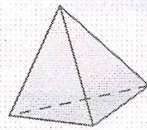**Related Words:** face, edge, vertex

Definition: A *polyhedron* is a space figure, or three-dimensional figure, whose surfaces are polygons.

Origin: The word *polyhedron* combines the Greek prefix *poli-*, meaning "many," and *hedron*, meaning "base."
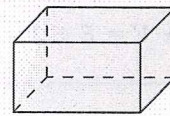
Examples: prism, pyramid
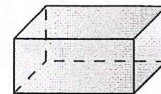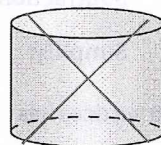
Non-Examples: circle, cylinder, sphere
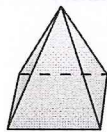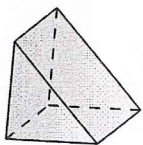
polyhedra

Pyramid   Prism
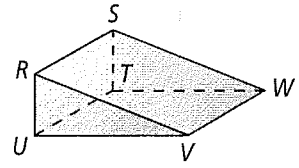
● Use Your Vocabulary

5. Cross out the figure below that is NOT a *polyhedron*.

Problem 1 Identifying Vertices, Edges, and Faces

Got It? How many vertices, edges, and faces are in the polyhedron at the right? List them.

6. Identify each description as a *vertex*, an *edge*, or a *face*.

a point where three or more edges intersect	polygon	a segment where two or more faces intersect
<u>vertex</u>	<u>face</u>	<u>edge</u>

7. List the vertices.

   R, S, T, U, V, W

8. List the edges. Remember to list the dashed hidden edges.

   $\overline{RS}$, $\overline{VW}$, $\overline{RU}$, $\overline{RV}$, $\overline{ST}$, $\overline{SW}$, $\overline{TU}$, $\overline{TW}$, $\overline{UV}$

9. List the faces. Remember to list the hidden faces.

   quadrilateral *RSTU*, quadrilateral *RSWV* , quadrilateral *TUVW*, $\triangle STW$, $\triangle RUV$

10. The polyhedron has 6 vertices, 9 edges, and 5 faces.
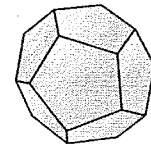

Key Concept   Euler's Formula

The sum of the number of faces (F) and vertices (V) of a polyhedron is two more than the number of its edges (E).

$$F + V = E + 2$$

Problem 2 Using Euler's Formula

Got It? Use Euler's Formula to find the number of faces for a polyhedron with 30 edges and 20 vertices.

11. Use the justifications at the right to find the number of faces.

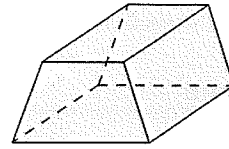|	
$F + V = E + 2$	Use Euler's Formula.
$F + \ 20 \ = \ 30 \ + 2$	Substitute with given information.
$F + \ 20 \ = \ 32$	Simplify.
$F = \ 32 \ - \ 20$	Subtraction Property of Equality
$F = \ 12$	Simplify.

12. A polygon with 30 edges and 20 vertices has 12 faces.

Lesson 11-1

Problem 3 Verifying Euler's Formula in Two Dimensions

Got It? Use the solid at the right. How can you verify Euler's Formula
$F + V = E + 2$ for the solid?

13. Count the number of vertices.

  4 on the bottom + 4 on the top = 8 vertices

14. Count the number of faces.

  2 bases + 4 lateral faces = 6 faces

15. Count the number of edges.

  9 solid edges + 3 dashed hidden edges = 12 edges

16. Now verify Euler's Formula for the values you found.

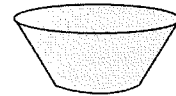|	
$F + V = E + 2$	Write Euler's Formula.
6 + 8 = 12 + 2	Substitute.
14 = 14	Simplify.

Problem 4 Describing a Cross Section

Got It? For the solid at the right, what is the cross section formed by
a horizontal plane?

Underline the correct word to complete each sentence.

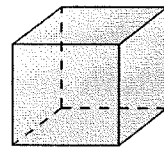**17.** A horizontal plane is parallel to the bottom / side of the solid.

18. A view from the side / top of the solid helps you see the shape of the cross section.

19. The cross section is a circle / trapezoid .

Problem 5 Drawing a Cross Section

Got It? Draw the cross section formed by a horizontal plane intersecting the left
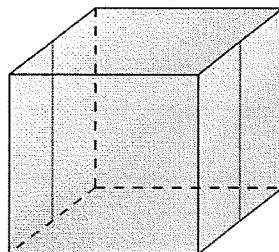and right faces of the cube. What shape is the cross section?
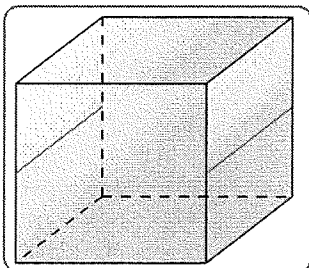
20. A *horizontal* plane is parallel to which faces of the cube? Circle your answer.
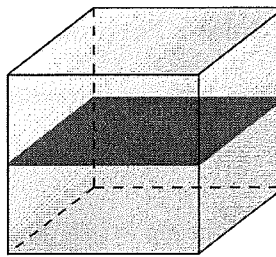
  front and back   left and right   top and bottom

21. Circle the diagram that shows the intersection of the horizontal plane and the left
and right faces of the cube.

22. Use the cube to draw and shade the cross section.

23. The cross section is a ?

square

Lesson Check • Do you UNDERSTAND?

Vocabulary Suppose you build a polyhedron from two octagons and eight squares. Without using Euler's Formula, how many edges does the solid have? Explain.

24. Complete the problem-solving model below.

Know	Need	Plan
The octagons and squares are ? .	To find the number of ? without using Euler's Formula	Count the number of edges formed by the squares and the top octagon.
faces	edges	Count the number of edges formed by the squares and the bottom octagon.
An edge is a segment formed by the intersection of two ? .		Count the number of edges formed by the squares.
faces		

25. The intersections of the squares and the top octagon form   8   edges.

26. The intersections of the squares and the bottom octagon form   8   edges.

27. The intersection of the eight squares form   8   edges.
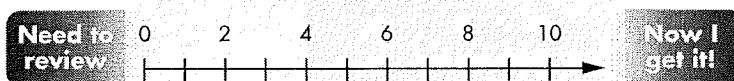
28. The solid has   8   +   8   +   8   =   24   edges.

Math Success

Check off the vocabulary words that you understand.

☐ polyhedron   ☐ face   ☐ edge   ☐ vertex   ☐ cross section
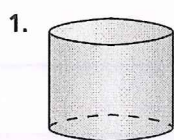
Rate how well you can *recognize polyhedra and their parts*.

| Need to review | 0 | 2 | 4 | 6 | 8 | 10 | Now I get it! |

Volumes of Prisms and Cylinders

Vocabulary

● Review

Label each diagram *cylinder* or *prism*.
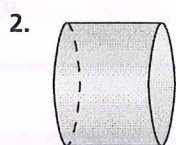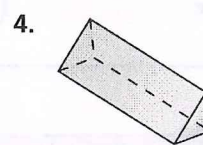
1. cylinder

2. cylinder
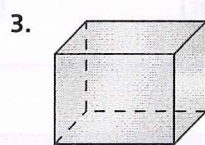
3. prism

4. prism

● Vocabulary Builder

composite (adjective, noun) **kum PAHZ it**

Related Words: compound, combination, component

Definition: Composite means put together with distinct parts.

Main Idea: A **composite** is a whole made up of different parts.

● Use Your Vocabulary

Complete each statement with the correct phrase from the list below. Use each phrase only once.

| *composite* function | *composite* map | *composite* number | *composite* sketch |

5. A __?__ combines different descriptions of features.

   composite sketch

6. A __?__ has factors other than one and the number.

   composite number

7. A __?__ shows the locations of shopping malls, houses, and roads in one illustration.

   composite map

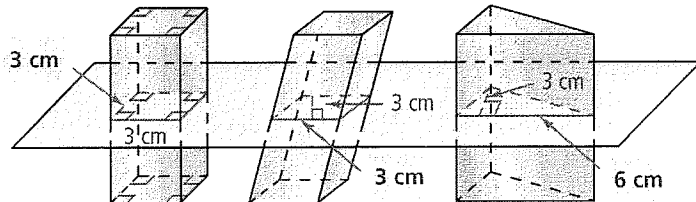8. A __?__ shows how to apply at least one function to another function.

   composite function

Theorem 11-5 Cavalieri's Principle

If two space figures have the same height and the same cross-sectional area at every level, then they have the same volume.

9. The three prisms below have the same height and the same volume. The first is a square prism. Label the missing dimensions.

10. Circle the solid(s) that may have the same cross-sectional area at every level.

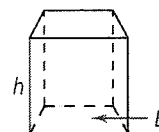cone   (cylinder)   (prism)   pyramid

Theorem 11-6 Volume of a Prism

The volume of a prism is the product of the area of the base and the height of the prism.

$V = Bh$

11. A prism with a base area of 15 m² and a height 4 m has a volume of **60** m³.

12. A prism with a volume of 81 ft³ and a height of 3 ft has a base area of **27** ft².

Problem 1 Finding the Volume of a Rectangular Prism

Got It? What is the volume of the rectangular prism at the right?

13. Circle the measurements of a base of the prism.

3 ft × 4 ft   (3 ft × 5 ft)   4 ft × 5 ft

14. Underline the correct word to complete the sentence.

The base is a <u>rectangle</u> / square .

15. Find B.

$B = \ell \cdot w$

$= 5 \cdot 3$

$= 15$

16. Find V.

$V = B \cdot h$

$= 15 \cdot 4$

$= 60$
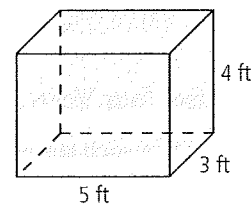
17. Underline the correct word to complete the sentence.

The units for this volume are <u>cubic</u> / square feet .

18. The volume of the prism is 60 ft³ .

Problem 2 Finding the Volume of a Triangular Prism

Got It? What is the volume of the triangular prism at the right?

19. The base is a right triangle with legs of length 10 m and 6 m.

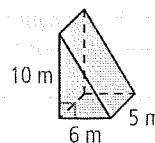20. The height of the prism is 5 m.

21. Complete the formula for volume of a prism. $V =$ B $\cdot$ h

22. Find the area of the base.

$A = \frac{1}{2} \cdot$ 6 $\cdot$ 10

$= \frac{1}{2} \cdot$ 60

$=$ 30

23. Find the volume of the prism.

$V =$ 30 $\cdot$ 5

$=$ 150

24. The volume of the triangular prism is 150 m^3.
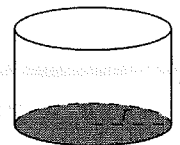
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take note

Theorem 11-7 Volume of a Cylinder

The volume of a cylinder is the product of the area of the base and the height of the cylinder.

$V = Bh$, or $V = \pi r^2 h$

25. Shade a base of the cylinder at the right. **Answers may vary. Sample:**

26. Describe the shape of the base.

The base is a circle.

Problem 3 Finding the Volume of a Cylinder

Got It? What is the volume of the cylinder at the right in terms of π?

27. Compete the reasoning model below.

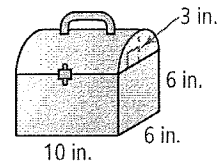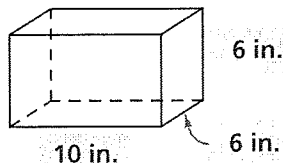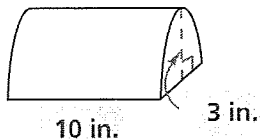| Think	Write
First I need to find the radius.	$r = \dfrac{2}{2} = $ 1
I can use the formula $V = \pi r^2 h$ and substitute for r and h.	$V = \pi \cdot$ 1 $^2 \cdot$ 3
Now I simplify.	$V = \pi \cdot$ 3
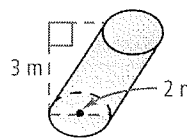
28. The volume of the cylinder is 3π m^3.

Got It? What is the approximate volume of the lunch box shown at the right? Round to the nearest cubic inch.

29. The top and bottom of the lunch box are sketched below. Label the dimensions.

10 in.   3 in.

6 in.

10 in.   6 in.

30. Find the volume of the top.

$V = \frac{1}{2}\pi r^2 h$

$= \frac{1}{2}\pi (\,3\,^2)(\,10\,)$

$= 45\pi$

31. Find the volume of the bottom.

$V = Bh$

$= (\,6\,\cdot\,6\,)(\,10\,)$

$= 360$

32. Find the sum of the volumes.

$V = \,45\,\pi + \,360$

$\approx \,501.3716694$

33. The approximate volume of the lunch box is 501 in.³.

Lesson Check • Do you UNDERSTAND?

Reasoning How is the volume of a rectangular prism with base 2 m by 3 m and height 4 m related to the volume of a rectangular prism with base 3 m by 4 m and height 2 m? Explain.

34. Cross out the formula that does NOT give the volume of a rectangular prism.

$V = Bh$ ~~$V = \pi r^2 h$~~ $V = \ell w h$

35. The Commutative / Identity Property of Multiplication states that the product of factors is the same when listed in a different order.

36. Now answer the question. **Explanations may vary. Sample:**

The volumes are equal by the Commutative Property of Multiplication.
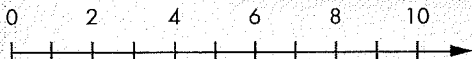
Math Success

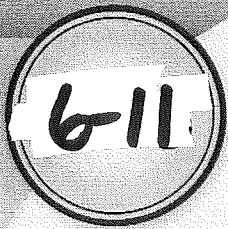Check off the vocabulary words that you understand.

☐ volume   ☐ composite space figure

Rate how well you can *find the volume of prisms and cylinders.*

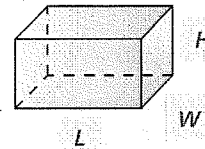Need to review   0   2   4   6   8   10   Now I get it!

6-11 Volumes of Pyramids and Cones

Vocabulary

Review

1. Write *L*, *W*, or *H* to label the *length, width,* and *height* of the rectangular prism at the right.

2. Explain how the *length, width,* and *height* of a cube are related.

   The length, width and height of a cube are equal.

Circle the correct statement in each exercise.

3. The *width* of a cylinder is the radius of a base of the cylinder.

   (The *height* of a cylinder is the *length* of an altitude of the cylinder.)

4. (The *height* of a pyramid is the length of a segment perpendicular to the base.)

   The slant *height* of a pyramid is the length of a segment perpendicular to the base.

Vocabulary Builder

volume (noun) VAHL **yoom**

Related Word: capacity

Main Idea: Volume measures quantity of space or amount, such as loudness of sound or a collection of books.

Definition: Volume is the amount of space that a three-dimensional figure occupies, measured in cubic units.

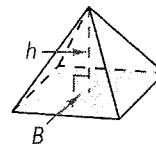**Example:** The **volume** of a bottle of juice is 2 liters.

Use Your Vocabulary

Write T for *true* or F for *false.*

T 5. A synonym for *volume* is capacity.

F 6. *Volume* is measured in square units.

F 7. You can find the *volume* of a circle.

Theorem 11-8 Volume of a Pyramid

The volume of a pyramid is one third the product of the area of the base and the height of the pyramid.

8. Complete the formula for the volume of a pyramid.

$V = \boxed{\frac{1}{3}} \cdot Bh$

Theorem 11-9 Volume of a Cone

The volume of a cone is one third the product of the area of the base and the height of the cone.
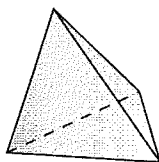
$V = \frac{1}{3}Bh$

9. Circle an equivalent formula for the volume of a cone.

$\boxed{V = \frac{1}{3}\pi r^2 h}$ $\qquad\qquad$ $V = \frac{1}{3} \cdot 2\pi r \cdot h$

Write the formula for the volume of each figure below.

10.

$V = \frac{1}{3} \cdot B \cdot h$

11.
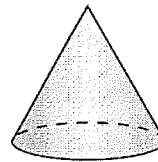
$V = \frac{1}{3}Bh$

Problem 1 Finding Volume of a Pyramid

Got It? A sports arena shaped like a pyramid has a base area of about 300,000 ft^2 and a height of 321 ft. What is the approximate volume of the arena?

12. Complete the problem-solving model below.

Know	Need	Plan
$B \approx \boxed{300,000}$ $h = \boxed{321}$	Volume of the pyramid	Substitute the given values into the formula $\boxed{V = \frac{1}{3}Bh}$.

13. Solve for V.

$V = \frac{1}{3}Bh$

$\approx \frac{1}{3}(300,000)(321)$

$\approx 32,100,000$

14. The approximate volume of the arena is 32,100,000 ft^3.

![check] **Problem 2** Finding the Volume of a Pyramid

Got It? What is the volume of a square pyramid with base edges 24 m and slant height 13 m?

15. Label the pyramid at the right.

13 m
24 m
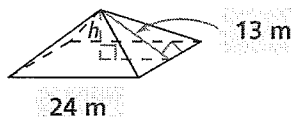
16. Find the height of the pyramid.

$13^2 = h^2 + \boxed{12}^2$

$169 = h^2 + \boxed{144}$

$h^2 = \boxed{25}$

$h = \boxed{5}$

17. Find the area of the base.

$B = \boxed{24} \cdot \boxed{24}$

$= \boxed{576}$

18. Find the volume of the pyramid.

$V = \boxed{\frac{1}{3}} \cdot Bh$

$= \boxed{\frac{1}{3}} \cdot \boxed{576} \cdot \boxed{5}$

$= \boxed{960}$

19. The volume of the pyramid is $\boxed{960}$ m³.

![check] **Problem 3** Finding the Volume of a Cone

Got It? A small child's teepee is 6 ft high with a base diameter of 7 ft. What is the volume of the child's teepee to the nearest cubic foot?

20. Label the cone at the right with the dimensions of the teepee.

21. The radius of the teepee is $\boxed{3.5}$ ft.

22. Use the justifications to find the volume of the teepee.

6 ft
7 ft

$V = \frac{1}{3}\pi r^2 h$ Use the formula with π for the volume of a cone.

$V = \boxed{\frac{1}{3}}\pi(3.5)^2(6)$ Substitute for r and h.

$= \boxed{\frac{1}{3}}\pi(12.25)(6)$ Square the radius.

$= \boxed{24.5}\pi$ Simplify in terms of π.

$\approx \boxed{76.96902001}$ Use a calculator.
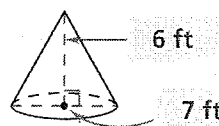
23. The volume of the child's teepee to the nearest cubic foot is $\boxed{77}$ ft³.

![check] **Problem 4** Finding the Volume of an Oblique Cone

Got It? What is the volume of the oblique cone at the right in terms of π and rounded to the nearest cubic meter?

24. The radius of the base is $\boxed{6}$ m and the height is $\boxed{12}$ m.

25. Cross out the formula that is NOT a formula for the volume of a cone.

12 m
6 m

$V = \frac{1}{3}Bh$ $\cancel{V = Bh}$ $V = \frac{1}{3}\pi r^2 h$

26. Find the volume of the cone.
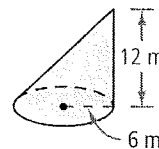
$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi (6)^2 (12)$$
$$= 144\pi$$
$$\approx 452.3893421$$

27. The volume of the cone in terms of π is 144π m³.

Rounded to the nearest cubic meter, the volume of the cone is 452 m³.

Lesson Check • Do you UNDERSTAND?

Error Analysis A square pyramid has base edges 13 ft and height 10 ft. A cone has diameter 13 ft and height 10 ft. Your friend claims the figures have the same volume because the volume formulas for a pyramid and a cone are the same: $V = \frac{1}{3}Bh$. What is her error?

28. Is $V = \frac{1}{3}Bh$ the volume formula for both a pyramid and a cone?   (Yes)/ No

Underline the correct word to complete each sentence.

29. The base of a square pyramid is a circle / polygon.

30. The base of a cone is a circle / polygon.

31. Circle the base used in the formula for the volume of a cone. Underline the base used in the formula for the volume of a square pyramid.

$$\boxed{B = \pi r^2} \qquad\qquad B = \frac{1}{2}bh \qquad\qquad \underline{B = s^2}$$

32. Now explain your friend's error. **Explanations may vary. Sample:**

She forgot that the area formulas for the

bases of pyramids and cones are different.

Math Success

Check off the vocabulary words that you understand.

☐ pyramid   ☐ cone   ☐ oblique   ☐ volume

Rate how well you can *find the volumes of pyramids and cones.*

Need to review 0   2   4   6   8   10 Now I get it!