Ms. Williams

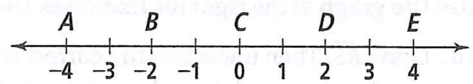
* Chapter 6 Notes *
Keys

6-1 Midsegments of Triangles

Vocabulary

Review

Use the number line at the right for Exercises 1–3.

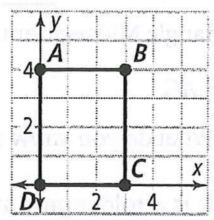


1. Point C is the *midpoint* of $\overline{AE}$.

2. Point D is the *midpoint* of $\overline{CE}$.

3. Point B is the *midpoint* of $\overline{AC}$.

Use the graph at the right for Exercises 4–6. Name each *segment*.



4. a *segment* that lies on the *x*-axis

   $\overline{DC}$ or $\overline{CD}$

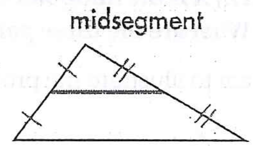
5. a *segment* that contains the point (0, 4)

   $\overline{DA}$, $\overline{AD}$, $\overline{AB}$, or $\overline{BA}$

6. a *segment* whose endpoints both have *x*-coordinate 3

   $\overline{BC}$ or $\overline{CB}$

Vocabulary Builder



midsegment (noun) MID seg munt

Related Words: midpoint, segment

Definition: A **midsegment** of a triangle is a segment connecting the midpoints of two sides of the triangle.

Use Your Vocabulary

Circle the correct statement in each pair.

7. A *midsegment* connects the midpoints of two sides of a triangle.

   A *midsegment* connects a vertex of a triangle to the midpoint of the opposite side.

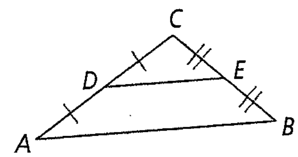
8. A triangle has exactly one *midsegment*. A triangle has three *midsegments.*

Theorem 5-1 Triangle Midsegment Theorem

take note

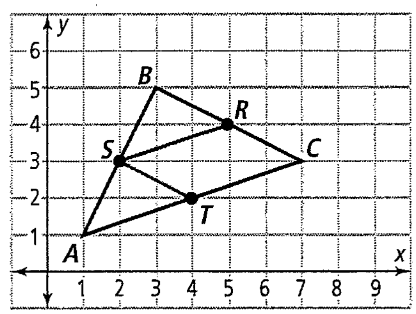
If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side and is half as long.

9. Use the triangle at the right to complete the table below.



If	Then
D is the midpoint of $\overline{CA}$ and	$\overline{DE} \parallel \overline{AB}$
E is the midpoint of $\overline{CB}$	$DE = \frac{1}{2}AB$

Use the graph at the right for Exercises 10–11.



10. Draw $\overline{RS}$. Then underline the correct word or number to complete each sentence below.

$\overline{RS}$ is a midsegment of / parallel to $\triangle ABC$.

$\overline{RS}$ is a midsegment of / parallel to $\overline{AC}$.

11. Use the Triangle Midsegment Theorem to complete.

$RS = \frac{1}{2}\ AC$

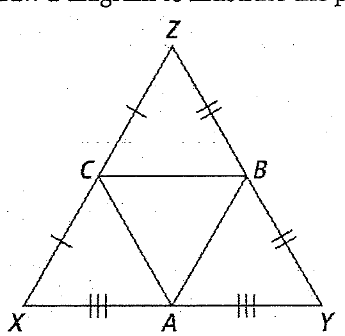
12. Draw $\overline{ST}$. What do you know about $\overline{ST}$?

Sample: It is a midsegment of $\triangle ABC$; it is parallel to and half the length of $\overline{BC}$.

Problem 1 Identifying Parallel Segments

Got It? **In $\triangle XYZ$, A is the midpoint of $\overline{XY}$, B is the midpoint of $\overline{YZ}$, and C is the midpoint of $\overline{ZX}$. What are the three pairs of parallel segments?**

13. Draw a diagram to illustrate the problem.

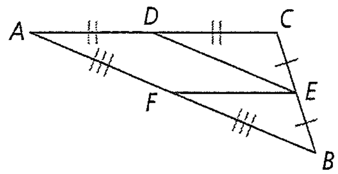


14. Write the segment parallel to each given segment.

$\overline{AB} \parallel \overline{ZX}$ $\qquad$ $\overline{CB} \parallel \overline{XY}$ $\qquad$ $\overline{CA} \parallel \overline{YZ}$

Problem 2 Finding Lengths

Got It? **In the figure below, $AD = 6$ and $DE = 7.5$. What are the lengths of $\overline{DC}$, $\overline{AC}$, $\overline{EF}$, and $\overline{AB}$?**



15. Complete the problem-solving model below.

Know	Need	Plan
$AD = 6$ and $DE = 7.5$. $CE = EB, AD = DC,$ $BF = \boxed{FA}$	The lengths of $\overline{DC}$, $\overline{AC}$, $\overline{EF}$, and $\overline{AB}$	Use the Triangle Midsegment Theorem to find DC, AC, EF, and $\boxed{AB}$.

16. The diagram shows that $\overline{EF}$ and $\overline{DE}$ join the midpoints of two sides of $\triangle ABC$.

By the Triangle Midsegment Theorem, $EF = \frac{1}{2} \cdot AC$ and $DE = \frac{1}{2} \cdot AB$.

Complete each statement.

17. $DC = AD = 6$

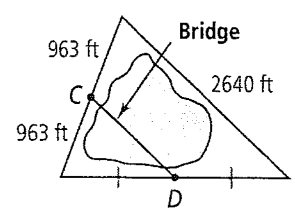
18. $AC = AD + DC = 6 + 6 = 12$

19. $EF = \frac{1}{2} \cdot AC = \frac{1}{2} \cdot 12 = 6$

20. $\cancel{CB}$ AB $= 2 \cdot DE = 2 \cdot 7.5 = 15$

Problem 3 Using the Midsegment of a Triangle

Got It? **$\overline{CD}$ is a bridge being built over a lake, as shown in the figure at the right. What is the length of the bridge?**



21. Complete the flow chart to find the length of the bridge.

$\overline{CD}$ joins the ? of two sides of a triangle. midpoints

$\overline{CD}$ is parallel to a side that is 2640 ft.

Use the Triangle ? Theorem. Midsegment

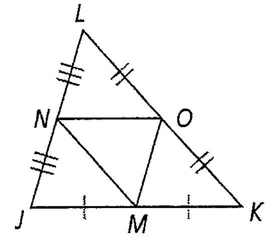
$CD = \frac{1}{2} \cdot 2640$

$CD = 1320$

22. The length of the bridge is 1320 ft.

Lesson Check • Do you know HOW?

If $JK = 5x + 20$ and $NO = 20$, what is the value of x?



Complete each statement.

23. N is the midpoint of $\overline{LJ}$.

24. O is the midpoint of $\overline{LK}$.

25. $\overline{NO}$ is a __?__ of $\triangle JKL$, so $NO = \frac{1}{2}JK$. midsegment

26. Substitute the given information into the equation in Exercise 25 and solve for x.

$NO = \frac{1}{2}JK$

$20 = \frac{1}{2}(2x + 20)$

$40 = (2x + 20)$

$20 = 5x$

$x = 4$

Lesson Check • Do you UNDERSTAND?

Reasoning **If two noncollinear segments in the coordinate plane have slope 3, what can you conclude?**

27. Place a ✓ in the box if the response is correct. Place an ✗ if it is incorrect.

✓ If two segments in a plane are parallel, then they have the same slope.

✗ If two segments lie on the same line, they are parallel.

28. Now answer the question. Answers may vary. Sample:

The segments do not lie on the same line,

so they are parallel lines.

Math Success

Check off the vocabulary words that you understand.

☐ midsegment ☐ midpoint ☐ segment

Rate how well you can *use properties of midsegments.*

Need to review 0 2 4 6 8 10 Now I get it!

5-2 Perpendicular and Angle Bisectors

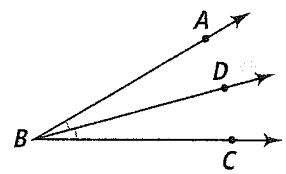
Vocabulary

Review

Complete each statement with *bisector* or *bisects*.

1. $\overrightarrow{BD}$ is the _?_ of $\angle ABC$. bisector

2. BD _?_ $\angle ABC$. bisects



Write T for *true* or F for *false*.

T 3. Two *perpendicular* segments intersect to form four right angles.

F 4. You can draw more than one line *perpendicular* to a given line through a point not on the line.

Vocabulary Builder

equidistant (adjective) ee kwih DIS tunt

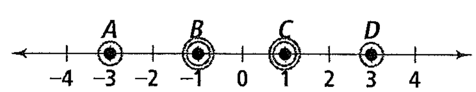
equidistant

Related Words: equal, distance

Definition: **Equidistant** means at an equal distance from a single point or object.

Use Your Vocabulary

Use to the number line at the right for Exercises 5 and 6.

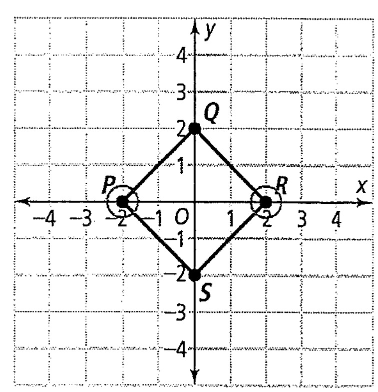


5. Circle two points *equidistant* from zero.

6. Name points that are *equidistant* from point C.

   B and D

Use to the diagram at the right for Exercises 7 and 8.



7. Circle two points *equidistant* from point Q.

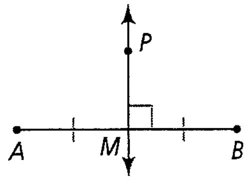
8. Name four segments that are *equidistant* from the origin.

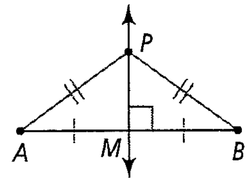
   $\overline{PQ}$ $\overline{QR}$ $\overline{RS}$ $\overline{SP}$

Take note: Theorem 5-2 Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

9. Use the diagrams below to complete the hypothesis and the conclusion.





If
$\overleftrightarrow{PM} \perp \overline{AB}$ and $AM =$ *MB*

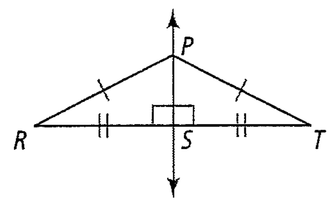
Then
$PA =$ *PB*

Take note: Theorem 5-3 Converse of the Perpendicular Bisector Theorem

10. Complete the converse of Theorem 5-2.

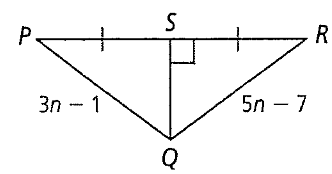
If a point is equidistant from the endpoints of a segment, then it is on the _?_ of the segment. perpendicular bisector

11. Complete the diagram at the right to illustrate Theorem 5-3.



Problem 1 Using the Perpendicular Bisector

Got It? **Use the diagram at the right. What is the length of $\overline{QR}$?**

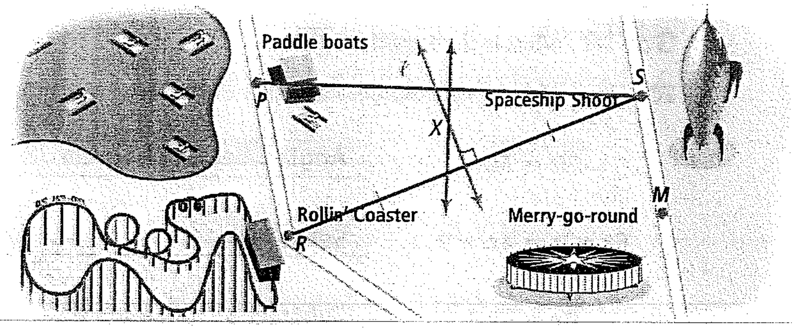


12. Complete the reasoning model below.

Think	Write
$\overline{QS}$ is the perpendicular bisector of $\overline{PR}$, so Q is equidistant from P and R by the Perpendicular Bisector Theorem.	$PQ = QR$ $3n - 1 = 5n - 7$
I need to solve for n.	$3n + 6 = 5n$ $6 = 2n$ $3 = n$
Now I can substitute for n to find QR.	$QR = 5n - 7$ $= 5(3) - 7 = 8$

Problem 2 Using a Perpendicular Bisector

Got It? **If the director of the park at the right wants a T-shirt stand built at a point equidistant from the Spaceship Shoot and the Rollin' Coaster, by the Perpendicular Bisector Theorem he can place the stand anywhere along line ℓ. Suppose the park director wants the T-shirt stand to be equidistant from the paddle boats and the Spaceship Shoot. What are the possible locations?**



13. On the diagram, draw $\overline{PS}$.

14. On the diagram, sketch the points that are equidistant from the paddle boats and the Spaceship Shoot. Describe these points.

the points on the perpendicular bisector of $\overline{PS}$

Got It? **Reasoning** **Can you place the T-shirt stand so that it is equidistant from the paddle boats, the Spaceship Shoot, and the Rollin' Coaster? Explain.**

15. Does the line you drew in Exercise 14 intersect line ℓ? Yes / No

16. Where should the T-shirt stand be placed so that it is equidistant from the paddle boats, the Spaceship Shoot, and the Rollin Coaster? Explain. **Answers may vary.**

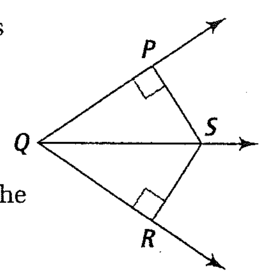
Sample: Place the stand at the intersection point X of the perpendicular bisectors of $\overline{RS}$ and $\overline{PS}$. By the Perpendicular Bisector Theorem, $XR = XS$ and $XS = XP$, so $XR = XS = XP$ by the Transitive Property.

The **distance from a point to a line** is the length of the perpendicular segment from the point to the line. This distance is also the length of the shortest segment from the point to the line.

Take note — Theorems 5-4 and 5-5

Angle Bisector Theorem

If a point is on the bisector of an angle, then the point is equisdistant from the sides of the angle.



17. If point S is on the angle bisector of $\angle$ PQR, then $SP =$ SR.

Converse of the Angle Bisector Theorem

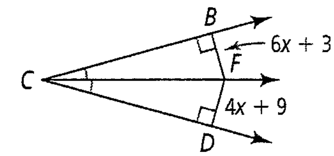
If a point in the interior of an angle is equidistant from the sides of the angle, then the point is on the angle bisector.

18. Point S is in the interior of $\angle$ PQR.

19. If $SP = SR$, then S is on the _?_ of $\angle PQR$. angle bisector

Problem 3 Using the Angle Bisector Theorem

Got It? What is the length of $\overline{FB}$?

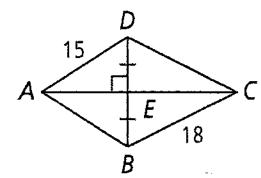


20. The problem is solved below. Justify each step.

$FB = FD$	Angle Bisector Theorem
$6x + 3 = 4x + 9$	Substitute.
$6x = 4x + 6$	Subtract 3 from each side.
$2x = 6$	Subtract $4x$ from each side.
$x = 3$	Divide each side by 2.
$FB = 6x + 3$	Given
$= 6(3) + 3 = 21$	Substitute 3 for x and simplify.

Lesson Check • Do you know HOW?

Use the figure at the right. What is the relationship between $\overline{AC}$ and $\overline{BD}$?



21. Underline the correct word or symbol to complete each sentence.

$\overline{AC}$ is parallel / <u>perpendicular</u> to $\overline{BD}$.

$\overline{AC}$ divides $\overline{BD}$ into two <u>congruent</u> / noncongruent segments.

$\overline{BD}$ divides $\overline{AC}$ into two congruent / <u>noncongruent</u> segments.

<u>$\overline{AC}$</u> / $\overline{BD}$ is the perpendicular bisector of $\overline{AC}$ / <u>$\overline{BD}$</u>.

Math Success

Check off the vocabulary words that you understand.

☐ perpendicular bisector ☐ equidistant ☐ distance from a point to a line

Rate how well you can *understand bisectors.*

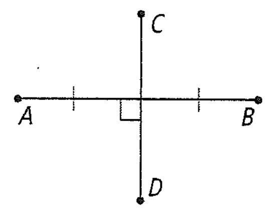
Need to review 0 2 4 6 8 10 Now I get it!

5-3 Bisectors in Triangles

Vocabulary

Review

Use the figure at the right. Write T for *true* or F for *false*.



F **1.** $\overline{AB}$ is the *perpendicular bisector* of $\overline{CD}$.

T **2.** $\overline{CD}$ is a *perpendicular bisector*, so it intersects $\overline{AB}$ at its midpoint.

T **3.** Any point on $\overline{CD}$ is *equidistant* from points A and B.

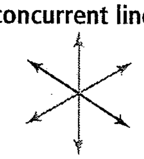
Vocabulary Builder

concurrent (adjective) **kun KUR unt**

Main Idea: Concurrent means occurring or existing at the same time.

Math Usage: When three or more lines intersect in one point, they are **concurrent.**

concurrent lines



Use Your Vocabulary

Complete each statement with *concurrency, concurrent,* or *concurrently.*

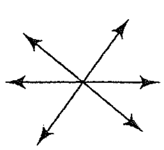
4. Two classes are __?__ when they meet at the same time. concurrent

5. The point of __?__ of three streets is the intersections of the streets. concurrency

6. A person may go to school and hold a job __?__. concurrently

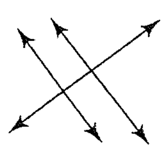
Label each diagram below *concurrent* or *not concurrent.*

7.



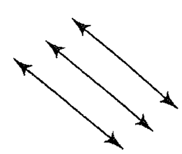
concurrent

8.



not concurrent

9.



not concurrent

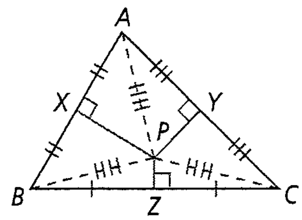
Theorem 5-6 Concurrency of Perpendicular Bisectors Theorem

take note

The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices.

Perpendicular bisectors $\overline{PX}$, $\overline{PY}$ and $\overline{PZ}$ are concurrent at P.

10. Mark $\triangle ABC$ to show all congruent segments.



Problem 1 Finding the Circumcenter of a Triangle

Got It? What are the coordinates of the circumcenter of the triangle with vertices $A(2, 7)$, $B(10, 7)$, and $C(10, 3)$?

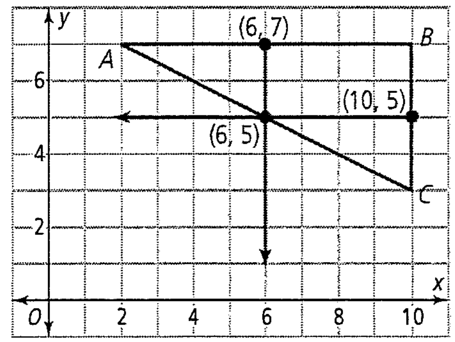
11. Draw $\triangle ABC$ on the coordinate plane.

12. Label the coordinates the midpoint of $\overline{AB}$ and the midpoint of $\overline{BC}$.

13. Draw the perpendicular bisector of $\overline{AB}$.

14. Draw the perpendicular bisector of $\overline{BC}$.

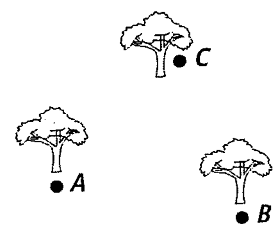
15. Label the coordinates of the point of intersection of the bisectors.



16. The circumcenter of $\triangle ABC$ is (6 , 5).

Problem 2 Using a Circumcenter

Got It? A town planner wants to place a bench equidistant from the three trees in the park. Where should he place the bench?



17. Complete the problem-solving model below.

Know	**Need**	**Plan**
The trees form the __?__ of a triangle.	Find the point of concurrency of the __?__ of the sides.	Find the __?__ of the triangle, which is equidistant from the three trees.
vertices	perpendicular bisectors	circumcenter

18. How can the town planner determine where to place the bench? Explain.

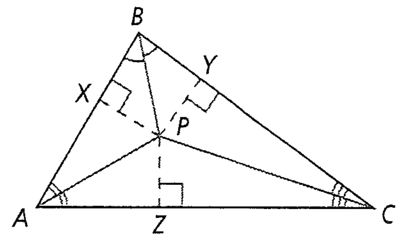
Explanations may vary. Sample: The town planner can place the bench at the circumcenter of the triangle formed by the three trees.

Take note

Theorem 5-7 Concurrency of Angle Bisectors Theorem

The bisectors of the angles of a triangle are concurrent at a point equidistant from the sides of the triangle.

Angle bisectors $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are concurrent at P.



19. $PX =$ PY = PZ

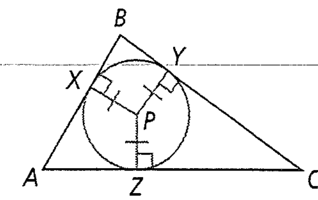
Complete each sentence with the appropriate word from the list.

incenter inscribed inside

20. The point of concurrency of the angle bisectors of a triangle is the _?_ of the triangle. incenter

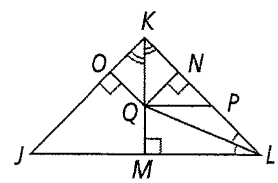
21. The point of concurrency of the angle bisectors of a triangle is always _?_ the triangle. inside

22. The circle is _?_ in $\triangle ABC$. inscribed



Problem 3 Identifying and Using the Incenter

Got It? $QN = 5x + 36$ and $QM = 2x + 51$. What is QO?



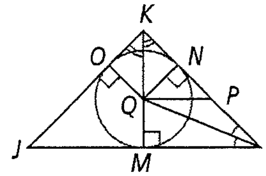
23. Complete the reasoning model below.

Think	Write
I know that Q is the point of concurrency of the angle bisectors.	Q is the incenter / midpoint of $\triangle JKL$.
And I know that	the distance from Q to each side of $\triangle JKL$ is equal / unequal .
I can write an equation and solve for x.	$QO = QM$ $5x + 36 = 2x + 51$ $5x = 2x + 15$ $3x = 15$ $x = 5$

24. Use your answer to Exercise 23 to find QO.

$QO = 5x + 36$
$QO = 5(5) + 36$
$QO = 25 + 36$
$QO = 61$

Got It? Reasoning **Is it possible for QP to equal 50? Explain.**



25. Drawn an inscribed circle in the diagram at the right.

26. $\overline{QN}$ and $\overline{QM}$ are two segments that have the same length as $\overline{QO}$.

27. Circle the correct relationship between QO and QP.

$QO < QP$ $\qquad$ $QO = QP$ $\qquad$ $QO > QP$

28. Given your answer to Exercise 27, is it possible for QP to equal 50? Explain. **Answers may vary. Sample:**

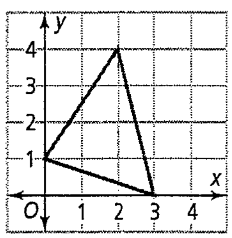
The radii measure 61, since $\overline{QO}$ is one of the radii. $\overline{QP}$ is longer than the radius of the circle. Therefore, its length cannot be 50.

Lesson Check • Do you UNDERSTAND?

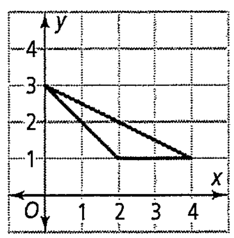
Vocabulary **A triangle's circumcenter is outside the triangle. What type of triangle is it?**

29. Draw an example of each type of triangle on a coordinate plane below. **Answers may vary. Samples are given.**

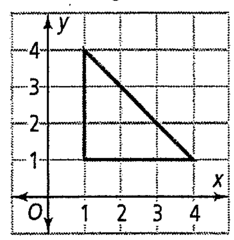
acute



obtuse



right



30. Circle the phrase that describes the circumcenter of a triangle.

the point of concurrency of the angle bisectors $\qquad$ the point of concurrency of the perpendicular bisectors of the sides

31. Underline the correct word to complete the sentence.

When a triangle's circumcenter is outside the triangle, the triangle is acute / obtuse / right.

Math Success

Check off the vocabulary words that you understand.

☐ concurrent ☐ circumscribed about ☐ incenter ☐ inscribed in ☐ bisector

Rate how well you can *use bisectors in triangles.*

Need to review 0 2 4 6 8 10 **Now I get it!**

5-4 Medians and Altitudes

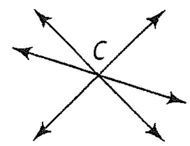
Vocabulary

Review

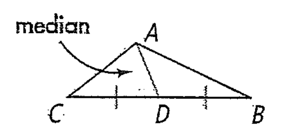
1. Are three diameters of a circle *concurrent*? Yes / No

2. Are two diagonals of a rectangle *concurrent*? Yes / No

3. Is point C at the right a point of *concurrency*? Yes / No



Vocabulary Builder



median (noun) MEE **dee un**

Related Words: median (adjective), middle (noun), midpoint (noun)

Definition: A **median** of a triangle is a segment whose endpoints are a vertex and the midpoint of the opposite side.

Use Your Vocabulary

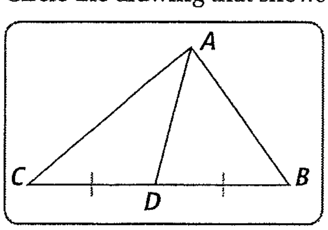
Write T for *true* or F for *false*.

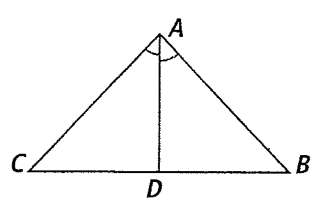
F 4. The *median* of a triangle is a segment that connects the midpoint of one side to the midpoint of an adjacent side.

T 5. The point of concurrency of the *medians* of a triangle is where they intersect.

F 6. A triangle has one median.

7. Circle the drawing that shows *median* $\overline{AD}$ of $\triangle ABC$.

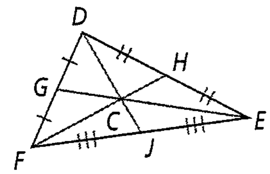




take note — Theorem 5-8 Concurrency of Medians Theorem

The medians of a triangle are concurrent at a point (the centroid of the triangle) that is two thirds the distance from each vertex to the midpoint of the opposite side.

For any triangle, the centroid is always inside the triangle.

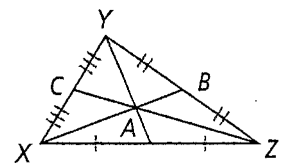


8. Complete each equation.

$DC = \frac{2}{3}\ DJ$ $\quad EC = \frac{2}{3}\ EG$ $\quad FC = \frac{2}{3}FH$

Problem 1 Finding the Length of a Median

Got It? **In the diagram at the right, $ZA = 9$. What is the length of $\overline{ZC}$?**



9. Point A is the centroid of $\triangle XYZ$.

10. Use the justifications at the right to solve for ZC.

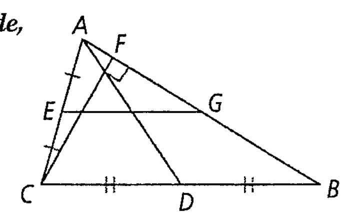
$ZA = \frac{2}{3} \cdot ZC$		Concurrency of Medians Theorem
$9 = \frac{2}{3} \cdot ZC$		Substitute for ZA.
$9\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)\frac{2}{3} \cdot ZC$		Multiply each side by $\frac{3}{2}$.
$\frac{27}{2} = ZC$		Simplify.

11. ZC is $\frac{27}{2}$, or $13\frac{1}{2}$.

An *altitude* of a triangle is the perpendicular segment from a vertex of the triangle to the line containing the opposite side.

Problem 2 Identifying Medians and Altitudes

Got It? **For $\triangle ABC$, is each segment, $\overline{AD}$, $\overline{EG}$, and $\overline{CF}$, a *median*, an *altitude*, or *neither*? Explain.**



12. Read each statement. Then cross out the words that do NOT describe $\overline{AD}$.

$\overline{AD}$ is a segment that extends from vertex A to $\overline{CB}$, which is opposite A.

$\overline{AD}$ meets $\overline{CB}$ at point D, which is the midpoint of $\overline{CB}$ since $\overline{CD} \cong \overline{DB}$.

$\overline{AD}$ is not perpendicular to $\overline{CB}$.

~~altitude~~ median ~~neither altitude nor median~~

13. Circle the correct statement below.

$\overline{AD}$ is a median. $\quad \overline{AD}$ is an altitude. $\quad \overline{AD}$ is neither a median nor an altitude.

14. Read the statement. Then circle the correct description of $\overline{EG}$.

$\overline{EG}$ does not extend from a vertex.

$\overline{EG}$ is a median. $\quad \overline{EG}$ is an altitude. $\quad \overline{EG}$ is neither a median nor an altitude.

15. Read each statement. Then circle the correct description of $\overline{CF}$.

$\overline{CF}$ is a segment that extends from vertex C to $\overline{AB}$, which opposite C.

$\overline{CF} \perp \overline{AB}$

$\overline{CF}$ is median. $\quad$ $\overline{CF}$ is an altitude. $\quad$ $\overline{CF}$ is neither a median nor an altitude.

Take note

Theorem 5-9 Concurrency of Altitudes Theorem

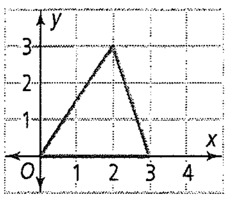
The lines that contain the altitudes of a triangle are concurrent.

The point of concurrency is the *orthocenter of the triangle.* The orthocenter of a triangle can be inside, on, or outside the triangle.

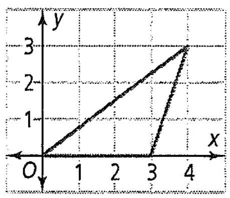
Answers may vary. Samples are given.

16. Draw an example of each type of triangle on a coordinate plane below.

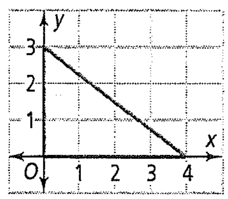
acute



obtuse



right



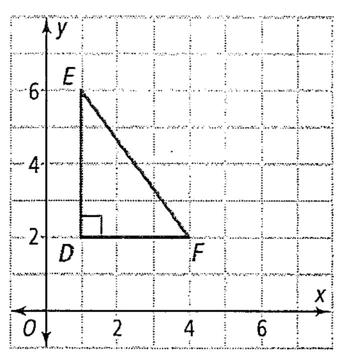
Draw a line from the type of triangle in Column A to the location of its orthocenter in Column B.

Column A	Column B
17. acute	outside the triangle
18. right	inside the triangle
19. obtuse	at a vertex of the triangle

Problem 3 Finding the Orthocenter

Got It? $\triangle DEF$ **has vertices** $D(1, 2)$, $E(1, 6)$, **and** $F(4, 2)$. **What are the coordinates of the orthocenter of** $\triangle DEF$**?**

20. Graph $\triangle DEF$ on the coordinate plane.



Underline the correct word to complete each sentence.

21. $\triangle DEF$ is a(n) acute / <u>right</u> triangle, so the orthocenter is at vertex D.

22. The altitude to $\overline{DF}$ is horizontal / <u>vertical</u>.

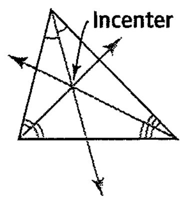
23. The altitude to $\overline{DE}$ is <u>horizontal</u> / vertical.

24. The coordinates of the orthocenter of $\triangle DEF$ are (1 , 2).

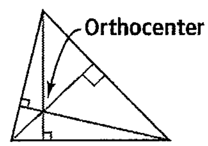
Concept Summary Special Segments and Lines in Triangles

take note

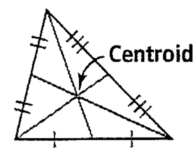
25. Use the words *altitudes, angle bisectors, medians,* and *perpendicular bisectors* to describe the intersecting lines in each triangle below.



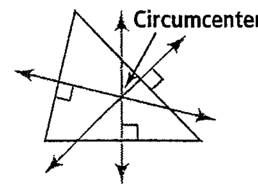
angle bisectors



altitudes



medians



perpendicular bisectors

Lesson Check • Do you UNDERSTAND?

Reasoning The orthocenter of $\triangle ABC$ lies at vertex A. What can you conclude about $\overline{BA}$ and $\overline{AC}$? Explain.

26. Circle the type of triangle whose orthocenter is located at a vertex.

acute right obtuse

27. $\overline{BA}$ and $\overline{AC}$ are sides of $\angle$ A.

28. Write your conclusion about $\overline{BA}$ and $\overline{AC}$. Justify your reasoning.

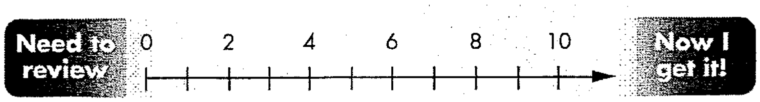
$\overline{BA}$ is perpendicular to $\overline{AC}$. Explanations may vary. Sample: $\triangle ABC$ is a right triangle and vertex A is a right angle.

Math Success

Check off the vocabulary words that you understand.

☐ median of a triangle ☐ altitude of a triangle ☐ orthocenter of a triangle

Rate how well you can *understand medians and altitudes.*



5-6 Inequalities in One Triangle

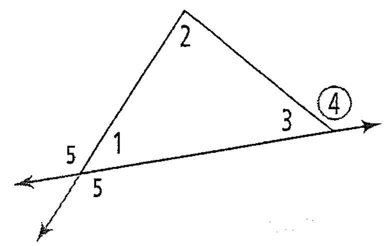
Vocabulary

Review

1. Circle the labeled *exterior angle*.
2. Write the *Exterior Angle* Theorem as it relates to the diagram.

   $m\angle$ 4 $= m\angle$ 1 $+ m\angle$ 2

3. Draw an *exterior angle* adjacent to $\angle 1$ and label it $\angle 5$.



Circle the statement that represents an *inequality* in each pair below.

4. $x \neq 32$

   $x = 32$

5. The number of votes is equal to 10,000.

   The number of votes is greater than 10,000.

Complete each statement with an inequality symbol.

6. *y* is less than or equal to *z*.

   $y \leq z$

7. The temperature *t* is at least 80 degrees.

   $t \geq 80°$

Vocabulary Builder

compare (verb) **kum** PEHR

There are more letters in the word *comparison* than in the word *compare*.

Other Word Form: comparison (noun)

Definition: To **compare** is to examine two or more items, noting similarities and differences.

Math Usage: Use inequalities to **compare** amounts.

Use Your Vocabulary

8. Complete each statement with the appropriate form of the word *compare*.

NOUN	By _?_, a spider has more legs than a beetle.	comparison
VERB	You can _?_ products before deciding which to buy.	compare
VERB	To _?_ quantities, you can write an equation or an inequality.	compare

Take note — Property: Comparison Property of Inequality

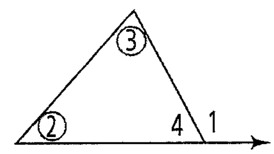
If $a = b + c$ and $c > 0$, then $a > b$.

9. Circle the group of values that satisfies the Comparison Property of Inequality.

$a = 5, b = 5$, and $c = 0$ $\qquad$ $a = 5, b = 2$, and $c = 3$ $\qquad$ $a = 8, b = 6$, and $c = 1$

Take note — Corollary: Corollary to the Triangle Exterior Angle Theorem

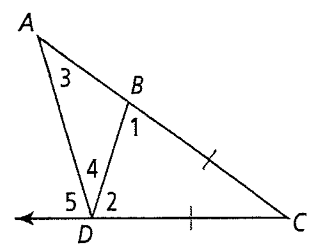
The measure of an exterior angle of a triangle is greater than the measure of each of its remote interior angles.



10. Circle the angles whose measures are always less than the measure of $\angle 1$.

Problem 1 Applying the Corollary

Got It? **Use the figure at the right. Why is $m\angle 5 > m\angle C$?**



Write the justification for each statement.

11. $\angle 5$ is an exterior angle of $\triangle ADC$.

Definition of an exterior angle

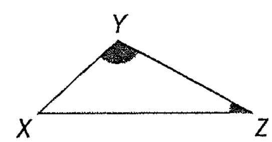
12. $m\angle 5 > m\angle C$

Corollary to the Triangle Exterior Angle Theorem

You can use the Corollary to the Triangle Exterior Angle Theorem to prove the following theorem.

Take note — Theorem 5-10 and Theorem 5-11

Theorem 5-10



If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side.

If $XZ > XY$, then $m\angle Y > m\angle Z$.

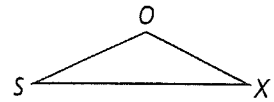
13. Theorem 5-11 is related to Theorem 5-10. Write the text of Theorem 5-11 by exchanging the words "larger angle" and "longer side."

Theorem 5-11 If two sides of a triangle are not congruent, then

the longer side lies opposite the larger angle.

Problem 3 Using Theorem 5-11

Got It? Reasoning **In the figure at the right, $m\angle S = 24$ and $m\angle O = 130$. Which side of $\triangle SOX$ is the shortest side? Explain your reasoning.**



14. By the Triangle Angle-Sum Theorem, $m\angle S + m\angle O + m\angle X = 180$, so $m\angle X =$ 180 $- m\angle S - m\angle O$.

15. Use the given angle measures and the equation you wrote in Exercise 14 to find $m\angle X$.

$m\angle X = 180 - 24 - 130 = 26$

16. Complete the table below.

angle	$\angle O$	$\angle X$	$\angle S$
angle measure	130	26	24
opposite side	$\overline{SX}$	$\overline{SO}$	$\overline{OX}$

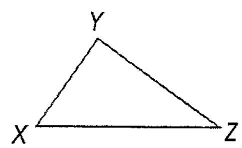
17. Which is the shortest side? Explain.

The shortest side is $\overline{OX}$ because it is opposite the smallest angle, $\angle$ S .

Take note

Theorem 5-12 Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



18. Complete each inequality.

$XY + YZ >$ XZ $\qquad YZ + ZX >$ YX $\qquad ZX + XY >$ ZY

Problem 4 Using the Triangle Inequality Theorem

Got It? **Can a triangle have sides with lengths 2 m, 6 m, and 9 m? Explain.**

19. Complete the reasoning model below.

Think	Write		
The sum of the lengths of any two sides must be greater than the length of the third side.	2 + 6 = 8	6 + 9 = 15	2 + 9 = 11
I need to write three sums and three inequalities.	8 < 9	15 > 2	11 > 6
One of those sums is greater / <u>not greater</u> than the length of the third side.	It is / <u>is not</u> possible for a triangle to have sides with lengths 2 m, 6 m, and 9 m.		

Problem 5 Finding Possible Side Lengths

Got It? **A triangle has side lengths of 4 in. and 7 in. What is the range of possible lengths for the third side?**

20. Let x = the length of the third side. Use the Triangle Inequality Theorem to write and solve three inequalities.

$x + 4 >$ 7 $\quad$ $x + 7 >$ 4 $\quad$ $7 + 4 >$ x

$x >$ 3 $\quad$ $x >$ -3 $\quad$ $11 >$ x

21. Underline the correct word to complete each sentence.

Length is always / sometimes / never positive.

The first / second / third inequality pair is invalid in this situation.

22. Write the remaining inequalities as the compound inequality 3 $< x <$ 11.

23. The third side must be longer than 3 in. and shorter than 11 in.

Lesson Check • Do you UNDERSTAND?

Error Analysis **A friend tells you that she drew a triangle with perimeter 16 and one side of length 8. How do you know she made an error in her drawing?**

24. If one side length is 8 and the perimeter is 16, then the sum of the lengths of the two remaining sides must be $16 - 8 =$ 8.

25. Underline the correct words or number to complete each sentence.

By the Triangle Inequality Theorem, the sum of the lengths of two sides of a triangle must be equal to / greater than / less than the length of the third side.

By the Triangle Inequality Theorem, the sum of the lengths of the two unknown sides must be equal to / greater than / less than the length 8 / 16.

But 8 is *not* equal to / greater than 8, so there must be an error in the drawing.

Math Success

Check off the vocabulary words that you understand.

☐ exterior angle ☐ comparison property of inequality

Rate how well you can ***use the Triangle Inequality Theorem.***

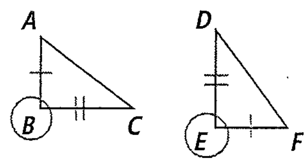
Need to review 0 2 4 6 8 10 Now I get it!

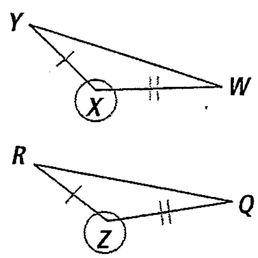
5-7 Inequalities in Two Triangles

Vocabulary

Review

Circle the *included angles* in each diagram.

1. 

2. 

In Exercises 3–5, cross out the group of values that does not satisfy the *Comparison Property of Inequality*.

3. ~~$a = 3, b = 3, c = 0$~~
$a = 6, b = 4, c = 2$

4. $a = 11, b = 3, c = 8$
~~$a = 1, b = 2, c = 3$~~

5. $a = 8, b = 3, c = 5$
~~$a = 8, b = 5, c = 4$~~

Write a number so that each group satisfies the *Comparison Property of Inequality*.

6. $a =$ 2 $, b = 0, c = 2$

7. $a = 9, b =$ 8 $, c = 1$

8. $a = 3, b =$ 1 $, c = 2$

Vocabulary Builder

hinge (noun, verb) hinj

Definition (noun): A **hinge** is a device on which something else depends or turns.

Definition (verb): To **hinge** upon means to depend on.

Use Your Vocabulary

Circle the correct form of the word *hinge*.

9. Everything *hinges* on his decision. Noun / Verb

10. The *hinge* on a gate allows it to swing open or closed. Noun / Verb

11. Your plan *hinges* on your teacher's approval. Noun / Verb

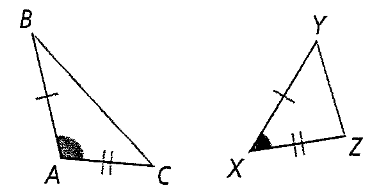
12. The lid was attached to the jewelry box by two *hinges*. Noun / Verb take note

Theorems 5-13 and 5-14 The Hinge Theorem and its Converse

If . . .	and . . .	then . . .	
two sides of one triangle are congruent to two sides of another triangle	the included angles are not congruent,	the longer third side is opposite the larger included angle.	Theorem 5-13 Hinge Theorem (SAS Inequality)
	the third sides are not congruent,	the larger included angle is opposite the longer third side.	Theorem 5-14 Converse of the Hinge Theorem (SSS Inequality)

13. Use the triangles at the right to complete the table.

Theorem	If...	Then...
5-13: Hinge Theorem	$m\angle A > m\angle X$	BC > YZ
5-14: Converse of the Hinge Theorem	$BC >$ YZ	$m\angle A >$ $m\angle X$



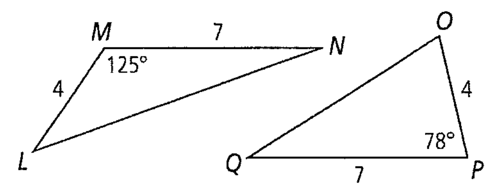
14. Explain why Theorems 5–13 and 5–14 are also called the SAS and SSS Inequality Theorems. **Answers may vary. Sample:**

Theorem 5-13 compares two pairs of sides and the pair of included angles (SAS).

Theorem 5-14 compares three pairs of sides (SSS).

Problem 1 Using the Hinge Theorem

Got It? What inequality relates LN and OQ in the figure at the right?



15. Use information in the diagram to complete each statement.

The included angle in $\triangle LMN$ is $\angle$ **M** .

The included angle in $\triangle OPQ$ is $\angle$ **P** .

16. Circle the side opposite the included angle in $\triangle LMN$. Underline the side opposite the included angle in $\triangle OPQ$.

$\overline{LM}$ $\overline{LN}$ $\overline{MN}$

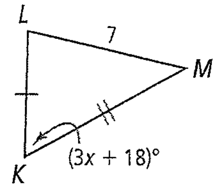
$\overline{QO}$ $\overline{QP}$ $\overline{OP}$

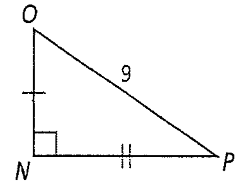
17. Use the Hinge Theorem to complete the statement below

$m\angle$ **M** $> m\angle$ **P** , so $LN >$ **OQ**.

Problem 3 Using the Converse of the Hinge Theorem

Got It? **What is the range of possible values for x in the figure at the right?**





18. From the diagram you know that the triangles have two pairs of congruent corresponding sides, that $LM <$ OP , and that $m\angle N =$ 90 .

Complete the steps and justifications to find upper and lower limits on x.

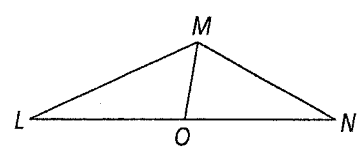
19. $m\angle K <$ $m\angle N$	Converse of the Hinge Theorem
$3x + 18 <$ 90	Substitute.
$3x <$ 72	Subtract 18 from each side.
$x <$ 24	Divide each side by 3 .
20. $m\angle K >$ 0	The measure of an angle of a triangle is greater than 0.
$3x + 18 >$ 0	Substitute.
$3x >$ -18	Subtract 18 from each side.
$x >$ -6	Divide each side by 3 .

21. Write the two inequalities as the compound inequality -6 $< x <$ 24 .

Problem 4 Proving Relationships in Triangles

Got It? **Given:** $m\angle MON = 80$; O is the midpoint of $\overline{LN}$.

Prove: $LM > MN$

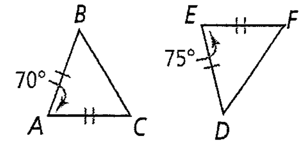


22. Write a justification for each statement.

Statements	Reasons
1) $m\angle MON = 80$	1) Given
2) $m\angle MON + m\angle MOL = 180$	2) Supplementary Angles
3) $80 + m\angle MOL = 180$	3) Substitute 80 for $m\angle MON$.
4) $m\angle MOL = 100$	4) Subtract 80 from each side.
5) $\overline{LO} \cong \overline{ON}$	5) O is the midpoint of $\overline{LN}$.
6) $\overline{MO} \cong \overline{MO}$	6) Reflexive Property of Congruence
7) $m\angle MOL > m\angle MON$	7) 100 > 80
8) $LM > MN$	8) Hinge Theorem

Lesson Check • Do you know HOW?

Write an inequality relating FD and BC.



In Exercises 23–26, circle the correct statement in each pair.

23. $\overline{AC} \cong \overline{EF}$ $AC > EF$ **24.** $AB > ED$ $\overline{AB} \cong \overline{ED}$

25. $m\angle BAC > m\angle FED$ $m\angle BAC < m\angle FED$

26. By the Hinge Theorem, you can relate FD and BC.

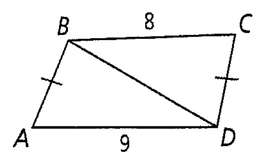
By the Converse of Hinge Theorem, you can relate FD and BC.

27. Write an inequality relating FD and BC.

$FD > BC$

Lesson Check • Do you UNDERSTAND?

Error Analysis **From the figure at the right, your friend concludes that $m\angle BAD > m\angle BCD$. How would you correct your friend's mistake?**



Write T for *true* or F for *false*.

T **28.** $AB = CD$ F **29.** $AD = CB$ T **30.** $BD = BD$

31. Your friend should compare AD and CB.

32. The longer of the two sides your friend should compare is AD.

33. How would you correct your friend's mistake? Explain.

Answers may vary. Sample: $AD > CB$, so use the Converse of the Hinge Theorem to conclude that $m\angle ABD > m\angle CDB$.

Math Success

Check off the vocabulary words that you understand.

☐ exterior angle ☐ comparison property of inequality ☐ Hinge Theorem

Rate how well you can *use triangle inequalities.*

Need to review 0 2 4 6 8 10 Now I get it!