

AFM - Final Exam Review

Name: Key

Functions:

Solve each equation. Remember to check for extraneous solutions.

1) $\frac{1}{a} - \frac{1}{a^2 - 5a} = \frac{4}{a^2 - 5a}$

2) $\frac{2}{p^2 - 2p + 1} = \frac{6}{p - 1} - \frac{1}{p^2 - 2p + 1}$

3) $\frac{n+5}{n} + \frac{5n-30}{n^2-n} = 1$

4) $\frac{1}{n^2-3n} = \frac{6n+1}{n-3} - 1$

OMIT

Solve each equation.

5) $\log_{11} 5v = \log_{11} (3v+9)$

$$5v = 3v + 9$$

$$2v = 9$$

$$v = 9/2$$

7) $\log_7 (x+1) + \log_7 x = \log_7 20$

$$\log_7 (x^2+x) = \log_7 20$$

$$x^2+x=20$$

$$x^2+x-20=0$$

$$(x-4)(x+5)$$

$$x=4$$

9) $7.1e^{x+7} = 20$

$$e^{x+7} = 20/7.1$$

$$\ln \frac{20}{7.1} = x+7$$

$$x = -5.904$$

11. Given the function: $f(x) = \begin{cases} 2x-5 & \text{if } x \leq 1 \\ 4-3x^2 & \text{if } x > 1 \end{cases}$

Find: $f(4) + 2f(-3) - 5f(1)$

$$-44 + 2(-11) - 5(-3) = -51$$

6) $\log_{16} (v^2+2v) = \log_{16} (54-v)$

$$v^2+2v=54-v$$

$$v^2+3v-54=0$$

$$(v+9)(v-6)=0$$

$$v = -9, 6$$

8) $\log_2 (x-4) - \log_2 7 = 3$

$$\log_2 \frac{x-4}{7} = 3$$

$$7 \cdot 8 = \frac{x-4}{7}$$

$$56 = x-4$$

$$x = 60$$

10) $4^{9x-6} + 1 = 9.2$

$$4^{9x-6} = 8.2$$

$$\log_4 8.2 = 9x-6$$

$$\frac{\log_4 8.2 + 6}{9} = x$$

$$x = .8353$$

Sequences & Series:

12. For the following arithmetic sequence, find the 18th term of 1.4, 1.9, 2.4

$$d = .5 \quad a_n = 1.4 + .5(n-1) \quad a_{18} = 1.4 + .5(18-1)$$

$$\boxed{a_{18} = 9.9}$$

13. Find the three arithmetic means between 2 and 5.

$$\underline{2}, \underline{2.75}, \underline{3.5}, \underline{4.25}, \underline{5} \quad d = \frac{5-2}{4} = .75$$

14. Find the sum of the first 35 terms of the arithmetic sequence when $a_1 = 5$ and $d = 4$

$$S = \frac{35}{2} (5 + 141) = \boxed{2555}$$

$$a_n = 5 + 4(n-1)$$

$$a_{35} = 5 + 4(34)$$

$$a_{35} = 141$$

15. Find the sum of the arithmetic series in which $a_1 = 5$ and $a_{34} = 71$

$$S = \frac{34}{2} (5 + 71) = \boxed{1292}$$

16. Evaluate: $\sum_{n=1}^4 (2n - 7)$

$$S = \frac{4}{2} (-5 + 1) = \boxed{-8}$$

17. Find the sixth term of the geometric sequence: $1, \frac{3}{4}, \frac{9}{16}, \dots$

$$r = 3/4 \quad a_n = 1 \left(\frac{3}{4} \right)^{n-1} \quad a_6 = \frac{243}{1024} = .237$$

18. Find the sixth term of the geometric sequence if $a_1 = 48$ and $r = -2$

$$a_n = 48(-2)^{n-1} \quad a_6 = -1536$$

19. Find the 8th term of the geometric sequence when $a_1 = 9$ and $r = -2$

$$a_n = 9(-2)^{n-1} \quad a_8 = -1152$$

20. Find the sum of the first five terms of the geometric series: $\frac{1}{3} + 2 + 12 + \dots$

$$r = 6$$

$$S = \frac{\frac{1}{3}(1-6^5)}{1-6} = \frac{1555}{3} = 518.33$$

21. Find the sum of the infinite geometric series, if it exists. $\sum_{n=1}^{\infty} 2k$

~~86~~ no sum

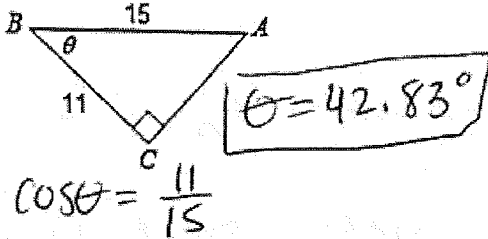
22. Find the sum of the infinite geometric series, if it exists: $20 - 2 + \frac{1}{5} - \dots$

$$r = \frac{-1}{10} \quad S = \frac{20}{1 - (-1/10)} = \frac{200}{11} = 18.18$$

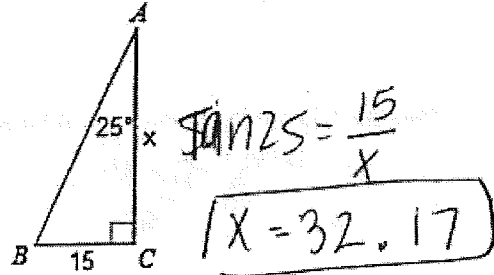
Trigonometry & Triangles:

23. Find the missing side or angle using trigonometry.

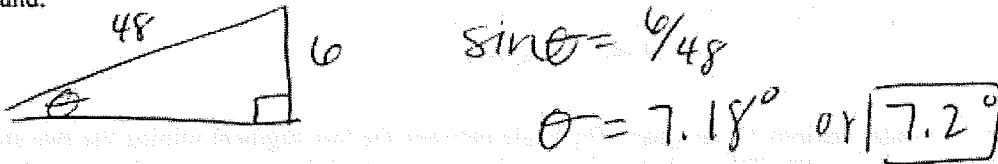
a.



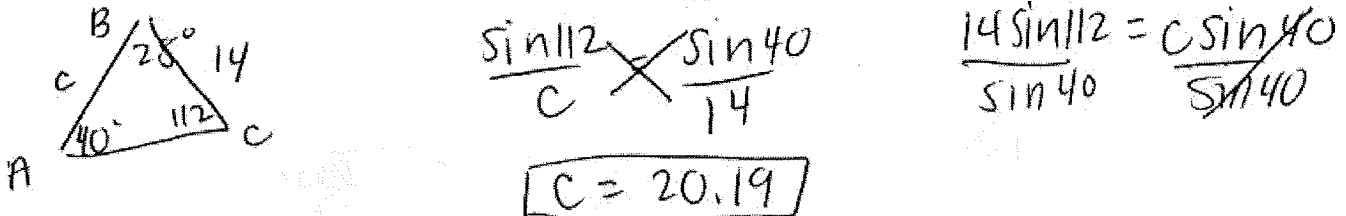
b.



24. A ramp in a park is 48 feet long and rises 6 feet. Estimate the angle to the nearest tenth that the ramp makes with the ground.



25. Given triangle ABC with $a = 14$, $A = 40^\circ$, and $B = 28^\circ$, what is the measure of c ?



26. Given triangle ABC with $a = 7$ cm, $b = 9$ cm, and $c = 14$ cm. Find the measure of angle C.

$$14^2 = 7^2 + 9^2 - 2(7)(9)\cos C$$

$$\cos C = \frac{14^2 - 7^2 - 9^2}{-2(7)(9)}$$

Take the Inverse

$C = 121.59^\circ$

27. Determine the number of triangles. (Use Law of Sines)

a. $m\angle B = 18^\circ$, $a = 10$ m, $b = 4$ m

$$\frac{\sin 18}{4} = \frac{\sin A}{10}$$

$$180 - 50.6 = 129.4$$

$$+ 18 = 147.4$$

2 D's

$A = 50.6^\circ$

b. $m\angle B = 78^\circ$, $b = 29$ ft, $a = 24$ ft

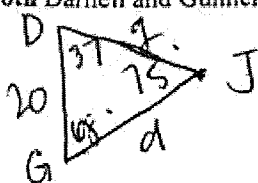
$$\frac{\sin 78}{29} = \frac{\sin A}{24}$$

$$180 - 54.05 = 125.95$$

$$+ 78 = 203.95$$

1 D

28. In a paintball game, there are only three players left. Darnell and Gunner are on the same side and 20 ft apart. Josh forms an angle of 75° between Gunner and Darnell. Gunner forms a 68° angle between Darnell and Josh. How far is Josh from both Darnell and Gunner?

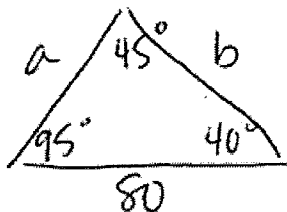


$$\frac{\sin 37}{d} = \frac{\sin 75}{20}$$

$d = 12.46$ ft from Gunner

$$\frac{\sin 68}{g} = \frac{\sin 75}{20}$$

$g = 19.2$ ft from Darnell



29. Rick and Carl want to plant corn along the edges of a triangular plot of land at the prison. Two of the angles of the triangle measure 95° and 40° . The side between these two angles is 80 feet long.

a. Find the measure of the third angle.

$$180 - 95 - 40 = 45^\circ$$

b. Find the length of the other two sides of the triangle.

$$\frac{\sin 40^\circ}{a} = \frac{\sin 45^\circ}{80}$$

$$80 \sin 40^\circ = a \sin 45^\circ$$

$$a = 72.72$$

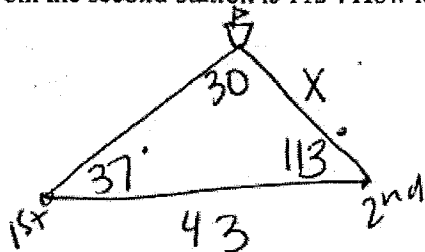
c. What is the perimeter of this triangular plot of land?

$$\frac{\sin 95^\circ}{b} = \frac{\sin 45^\circ}{80}$$

$$b = 112.71$$

$$80 + 112.71 + 72.72 = 265.43 \text{ ft}$$

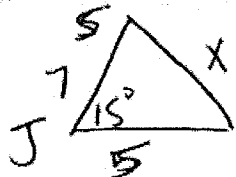
30. A ship is sighted from two radar stations 43 km apart. The angle between the line segment joining the two stations and the radar beam of the first station is 37° . The angle between the line segment joining the two stations and the beam from the second station is 113° . How far is the ship from the second station?



$$\frac{\sin 30^\circ}{43} = \frac{\sin 37^\circ}{X}$$

$$X = 51.76 \text{ km}$$

31. During a figure skating routine, Jessica and Shannon skate apart with an angle of 15° between them. Jessica skates for 5 meters and Shannon for 7 meters. How far apart are the skaters?



$$X^2 = 7^2 + 5^2 - 2(7)(5)\cos 15^\circ$$

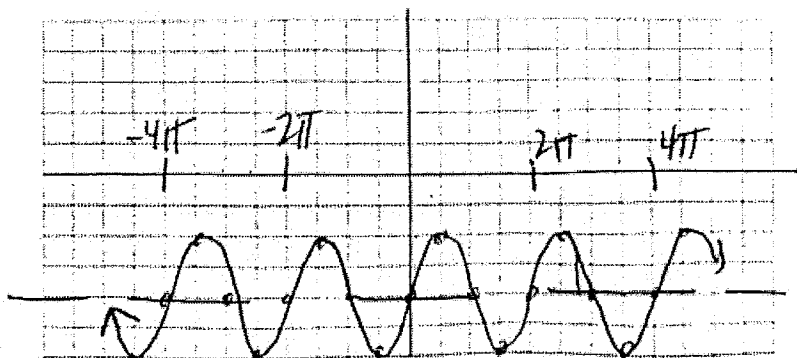
$$X = 2.527 \text{ m}$$

32. Graph $y = 2 \sin \theta - 4$

$$a = 2$$

$$p = 2\pi$$

up: Down 4



33. State the amplitude, period, vertical shift, and horizontal shift for: $y = 6 \sin(4\theta) + 5$

$$a = 6 \quad p = \frac{2\pi}{4} = \frac{\pi}{2} \quad \text{v. shift} \rightarrow \text{up } 5 \\ \text{no horiz. shift}$$

Statistics:

For questions 19 – 21, use the following information: On a normal curve, the mean on the Algebra II Final is 54, with a standard deviation of 11.8

34. What percent of students are within 2 standard deviations of the mean?

95%

35. If 120 students took the test, how many scored higher than 65.8?

$$\text{normalcdf}(65.8, 1E99, 54, 11.8) = 0.15806 (120) = 19 \text{ students}$$

36. What percent scored lower than 54?

$$\text{normalcdf}(-1E99, 54, 54, 11.8) \\ = 0.5 \text{ or } 50\%$$

Probability:

37. A 9-member committee is selecting a president, vice-president, secretary, and treasurer from the committee. No person can serve in two positions. In how many ways can the four positions be filled?

$$9P_4 = 3024$$

38. Teddy is buying a sports car. He can buy red or black, convertible or hard-top, straight drive or automatic. How many possible models does he have to choose from?

$$2 \cdot 2 \cdot 2 = 8$$

39. How many possible ways can you choose 3 library books to check out from 8?

$$8C_3 = \frac{8!}{5!3!} = 56$$

40. How many ways can you arrange 8 candles on the top of a birthday cake?

diff candles,
in a line

$$8P_8 = 8! = 40320$$

41. How many ways are there to arrange the letters in "Trigonometry"?

$$\frac{12!}{2!2!2!} = 59875200 \quad \begin{matrix} 2 \text{ O's} & 2 \text{ r's} \\ 2 \text{ T's} \end{matrix}$$

42. How many ways can Mrs. Smith's preschool class of 12 students line-up to go outside and play?

$$12P_{12} = 12! = 479,001,600$$

43. How many groups of 5 students can be chosen from 25?

$${}_{25}C_5 = \frac{25!}{20!5!} = 53130$$

44. How many ways can you choose a group of 5 men and 7 women from 12 men and 13 women?

$${}_{12}C_5 \cdot {}_{13}C_7 = 1,359,072$$

45. Suppose you select 3 letters from the word CLEMSON. What is the probability of selecting 2 vowels at the same time?

$$\frac{2}{7} \cdot \frac{1}{6} \cdot \frac{5}{5} = \frac{1}{21} = .0476$$

Vowel vowel not vowel

46. A bag contains 8 orange marbles and 5 purple marbles. If a marble is chosen at a random, what is the probability that it is not purple?

$$\frac{8}{13}$$

47. Billy breaks his piggy bank and finds 5 pennies, 8 nickels, and 9 dimes. What is the probability that he will selection 1 dime and 1 nickel at the same time?

$$\frac{9}{22} \cdot \frac{8}{21} = \frac{12}{77} = .1558$$

48. What is the probability he will select 2 pennies at the same time?

$$\frac{5}{22} \cdot \frac{4}{21} = \frac{10}{231} = .0433$$

49. A die is thrown twice. What is the probability that a 4 is thrown followed by a 6?

$$\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

For questions 35 – 37, 12 playing cards (3 Aces, 4 Kings, 2 Queens, and 3 Jacks) are placed on the table face down. If four cards are selected at random, find the probability that:

50. You select Ace, Jack, King, King, without replacement.

$$\frac{3}{12} \cdot \frac{3}{11} \cdot \frac{4}{10} \cdot \frac{3}{9} = \frac{1}{110} = .0091$$

51. You select a Queen, King, Jack, Ace, with replacement.

$$\frac{2}{12} \cdot \frac{4}{12} \cdot \frac{3}{12} = \frac{1}{12} = .0833$$

52. You select Queen, Queen, Ace, any card other than Ace, without replacement.

$$\frac{2}{12} \cdot \frac{1}{12} \cdot \frac{7}{12} = \frac{7}{864} = .0081$$

Mixed Review – Multiple Choice:

53. Find the value of ${}_6P_4$.

$$\frac{6!}{2!} = 360$$

- a) 15 b) 2 c) 24 **d) 360**

54. Find the standard deviation for the given data: 5, 6, 8, 11, 10

- a) 3.28 b) 1.28 **c) 2.28** d) 4.28

55. Solve: $\frac{x+9}{x+8} = \frac{x-7}{x-6}$

$$(x+9)(x-6) = (x+8)(x-7)$$

$$x^2 + 3x - 54 = x^2 + x - 56$$

$$2x = -2$$

$$x = -1$$

- a) x = -1** b) x = 0 c) x = 2 d) x = -3

56. Evaluate: $\log_9 729$

$$9^3 = 729$$

- a) 3** b) 5 c) 4 d) 2

57. Evaluate: $\log 94$

$$10^? = 94$$

- a) 9.4 **b) 1.97** c) .51 d) 3.95

58. Solve $e^{4x} = 5.7$ for x to four decimal places.

$$\frac{\ln 5.7}{4} = x$$

- a) -0.4030 **b) 0.4351** c) 0.7559 d) -0.7559

59. $\log_9(x^2 + 7) = \log_9(43)$

$$x^2 + 7 = 43 \quad x^2 = 36 \quad x = \pm 6$$

- a) ± 36 **b) ± 6** c) ± 6.56 d) ± 5

60. $\ln(-2y + 5) - \ln(y + 4) = \ln(-11y - 2)$

- a) (-3.68, - .32)** b) infinite solutions c) (3.68, .32) d) no solution

$$\ln\left(\frac{-2y+5}{y+4}\right) = \ln(-11y-2)$$

$$\frac{-2y+5}{y+4} = (-11y-2)$$

$$-2y+5 = -11y^2 - 44y - 2y + 8$$

$$11y^2 + 44y + 13 = 0$$

$$x = \frac{-44 \pm \sqrt{44^2 - 4(11)(13)}}{2(11)}$$

$$x = -3.21, -3.679$$

61. Find an exponential function to model the data.

- a) $f(x) = 116.4 - 42.8 \ln x$
- ~~b) $f(x) = 2.204(3.56)^x$~~
- c) $f(x) = 3.56(2.04)^x$ $Y^2 = 9.99$
- d) $f(x) = -42.8 + 116.4 \ln x$

x	y
1	7
2	16
3	30
4	61
5	124
6	271
7	522

Enter in Data to List + Spreadsheets
* Calc Expo/Log Regression

62. Find the best fit regression model for the data according to the given model.

x	y
1	50
2	140
3	260
4	400
5	560
6	750
7	925
8	1130

- a) $49.79x^{1.50}$
- b) $5.48x^{32}$
- c) $156.13x - 175.71$
- d) $1.5x + 3.91$

63. What is the explicit form of the equation: $a_1 = a_{n-1} + 2(n-1)$; $a_1 = 1$

- A $a_n = 2n - 1$
- B $a_n = n^2 - n + 1$
- C $a_n = n^2 - 2n + 2$
- D $a_n = 2n^2 - 2n - 1$

64. Which function has an amplitude that is twice the size and a period that is three times the size of the function $y = 3 \cos\left(\frac{x}{4} - 1\right) + 2$

$P = \frac{2\pi}{1/4} = 8\pi$

A) $y = 6 \sin\left(\frac{x}{12} - 3\right) + 1$

~~B) $y = \frac{3}{2} \cos\left(\frac{3x}{4} + 1\right) - 3$~~

C) $y = 6 \cos\left(\frac{3x}{4} - 1\right) + 3$

~~D) $y = \frac{3}{2} \sin\left(\frac{x}{12} + 3\right) - 1$~~

$a = 6$

$P = 34\pi$

$\frac{2\pi}{1/12} = 24\pi$