Name_____ Class_____ Date_____

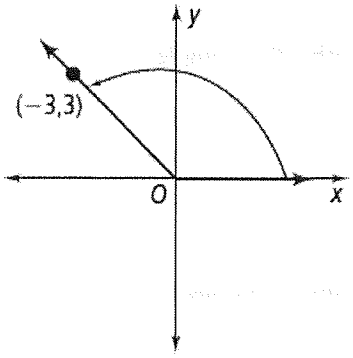
4-1 Practice

Form K

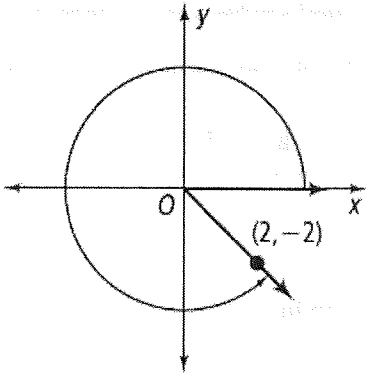
Angles and the Unit Circle

Find the measure of each angle in standard position.

1.



2.

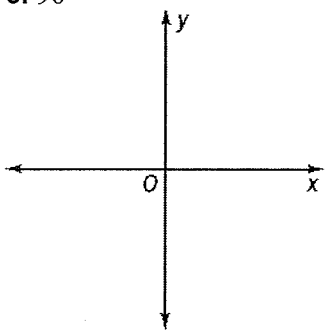


The terminal side of this angle passes through point (−3, 3), so it is 45° beyond a right angle.

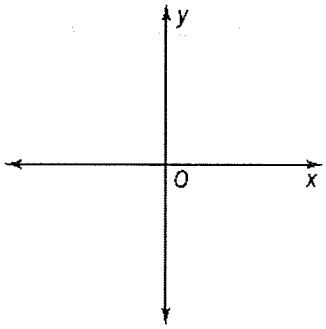
90° + 45° = ☐

Draw a sketch of each angle in standard position. Remember, the measure of an angle is positive when the angle opens in a counterclockwise direction. The measure is negative when the angle opens in a clockwise direction.

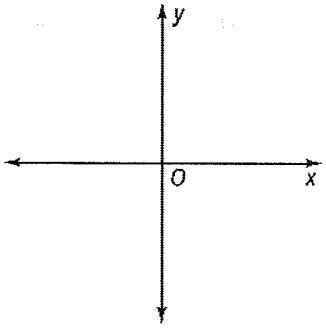
3. 90°



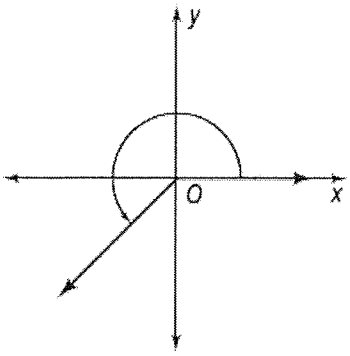
44. 225°



5. −135°



6. Error Analysis Your classmate believes that the angle shown below measures −225°. What error did your classmate make? What is the correct measure of the angle?



Name_____ Class_____ Date_____

4-1 Practice (continued)

Form K

Angles and the Unit Circle

Use your knowledge of coterminal angles to answer the following questions. Remember, coterminal angles share a terminal side.

7. Which of the following angles is not coterminal with the other three angles?

(A) 210° (B) −150° (C) 150° (D) 570°

8. Which of the following angles is not coterminal with the other three angles?

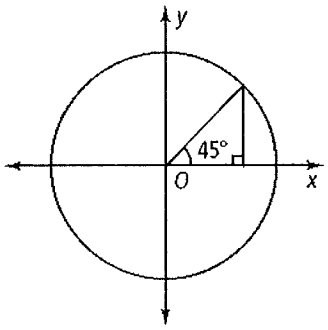
(F) 165° (G) 555° (H) 195° (I) −525°

Use a unit circle to find the sine and the cosine of the following angles.

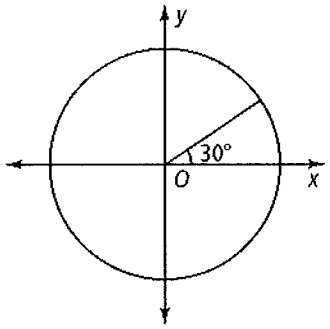
9. −270° **10.** 180° **11.** 360°

Find the exact sine and cosine of the following angles.

12.



13.



Remember, the lengths of the legs of a 45°–45°–90° triangle are $\frac{\sqrt{2}}{2}$ times the length of the hypotenuse.

Name_____ Class_____ Date_____

4-2 Practice

Form K

Radian Measure

Find the measure of each angle in radians.

1. $30°$

$30° \cdot \frac{\pi \text{ radians}}{180°}$

2. $140°$

3. $300°$

4. $15°$

5. $60°$

6. $260°$

Find the measure of each angle in degrees.

7. 2π radians

$2\pi \cdot \frac{180°}{\pi \text{ radians}}$

8. $\frac{2\pi}{3}$ radians

9. $\frac{\pi}{4}$ radians

Find the exact values of sin θ and cos θ for the following angles.

10. π

11. $\frac{3\pi}{4}$

12. $\frac{4\pi}{3}$

13. $\frac{\pi}{3}$

14. $\frac{11\pi}{6}$

15. $\frac{7\pi}{6}$

4-2 Practice (continued)

Form K

Radian Measure

16. Reasoning Why are radian angle measures sometimes more useful than degree measures?

Find the length of an arc of a circle, given the radius and angle measure.

17. radius = 4

$\theta = \frac{\pi}{2}$

$s = r\theta$

$s = 4\left(\frac{\pi}{2}\right)$

$s = 2\pi$

$s \approx$ ☐

18. radius = 7

$\theta = \frac{3\pi}{20}$

19. A large pizza with diameter of 18 in. is cut into 8 equal slices. How long is the crust of one slice of pizza?

20. Writing When does it make sense to keep your answers in terms of π? When do you need to simplify?

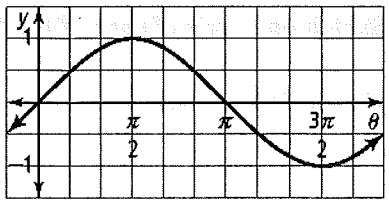
Name_____ Class_____ Date_____

4-3 Practice

Form K

The Sine Function

Use the graph at the right to find or estimate the value of $y = \sin \theta$ for each value of θ.



1. π radians

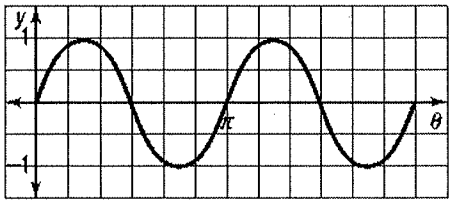
2. 2 radians

3. $\frac{3\pi}{2}$ radians

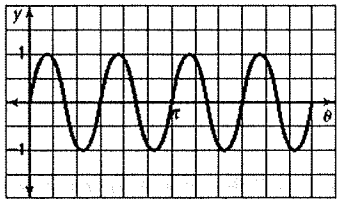
4. 4 radians

Determine the number of cycles each sine function has in the interval from 0 to 2π. Then find the period of each function.

5.

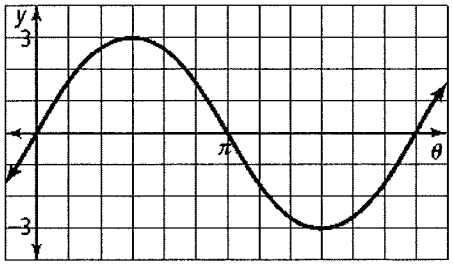


6.

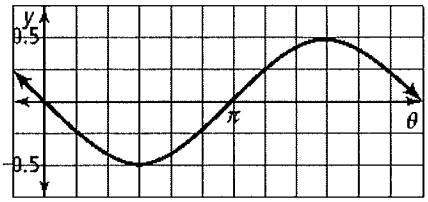


Find the amplitude of the following sine curves.

7. $y = 3 \sin x$



8. $y = -0.5 \sin x$



9. Error Analysis Your friend said that the amplitude of $y = -4 \sin x$ is -4. What error did she make? What is the correct amplitude?

4-3 Practice (continued)

Form K

The Sine Function

Sketch one cycle of each of the following sine curves. Assume $a > 0$.

10. amplitude 1, period 2π

11. amplitude 3, period 4π

12. amplitude 1.5, period π

13. amplitude 2, period 6π

Sketch one cycle of the graph of each sine function.

14. $y = 4 \sin 2\theta$

amplitude: 4
cycles from 0 to 2π : 2
period: π

15. $y = -3 \sin 4\theta$

Name _____ Class _____ Date _____

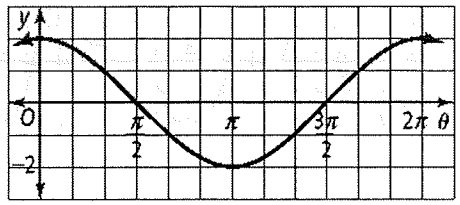
4-4 Practice

Form K

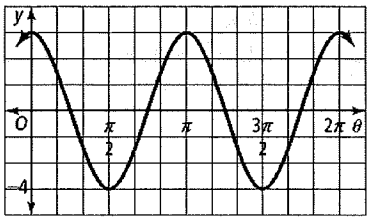
The Cosine Function

Find the period and the amplitude of each cosine function. Then determine where the maximum values, minimum values, and zeros occur in the interval from 0 to 2π.

1.



2.



Sketch the graph of each function in the interval from 0 to 2π.

3. $y = 3 \cos 2\theta$

amplitude: 3
cycles: 2
period: π

4. $y = 0.5 \cos \theta$

5. $y = -2 \cos 4\theta$

6. $y = \cos 2\theta$

Name_____ Class_____ Date_____

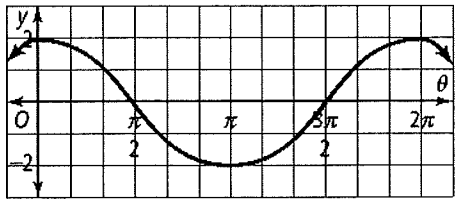
4-4 Practice (continued)

Form K

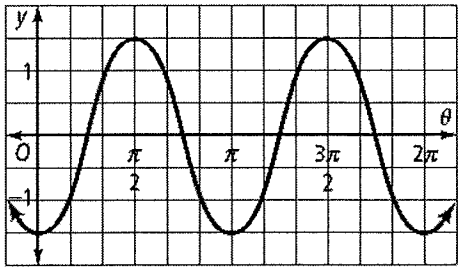
The Cosine Function

Write an equation of a cosine function for each graph.

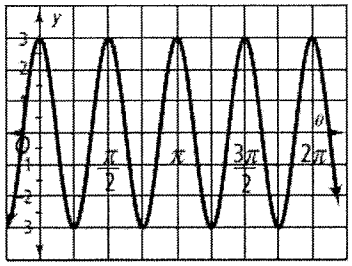
7.



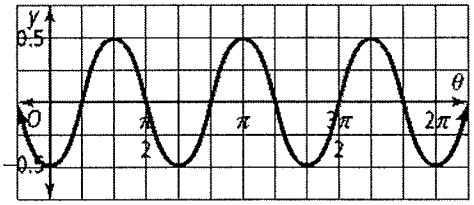
8.



9.



10.



Solve each equation in the interval from 0 to 2π. Round your answers to the nearest hundredth.

11. $\cos 2t = 0.25$

12. $5 \cos t = 1$

13. $4 \cos \frac{t}{4} = 2$

14. $-3 \cos t = 2.5$

15. $2 \cos 0.5t = -0.5$

16. $-0.5 \cos 2t = 0.3$

Name_____ Class _____ Date_____

Practice

Unit 4

Lessons 4-3 through 4-5

Identify the amplitude or asymptotes, and the period for each function.

1. $y = 4 \sin 3x$

2. $y = \cos 4x$

3. $y = \frac{1}{3} \tan \pi x$

4. $y = 2 \cos \frac{x}{4}$

5. $y = 3 \tan x$

6. $y = \frac{1}{9} \sin 5x$

Sketch the graph of one cycle of each function.

7. $y = 2 \cos x$

8. $y = 3 \sin 2x$

9. $y = \tan \frac{x}{2}$

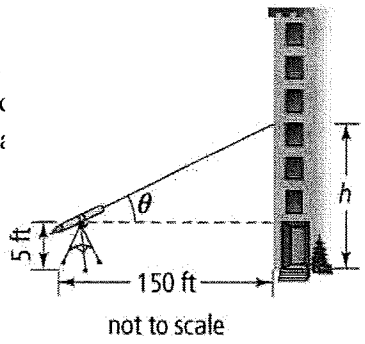
10. $y = -\sin 3x$

11. $y = \cos 4x$

12. $y = -2 \tan \pi x$

13. Earthquakes under the ocean can sometimes cause a dangerous wave called a tsunami. You can model the motion of a tsunami with the function $f(x) = a \cos bx$. Write an equation that models a tsunami that travels at 120 ft/s, has a period of 20 s, and has an amplitude of 60 ft.

14. Owen is using a telescope to measure a tall building down the telescope, Owen can see the point on the building that ground. The relationship between the position of the telesc can be modeled by $h = 150 \tan \theta + 5$. Owen changed the a 75°. How much higher is the point on the building that he



not to scale

Name_____ Class_____ Date_____

4-6 Practice

Form K

Translating Sine and Cosine Functions

Determine the value of h in each translation. Then describe each phase shift.

1. $g(x) = f(x - 5)$

2. $y = \sin(x - 3)$

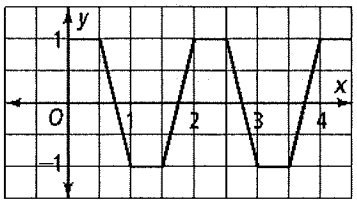
3. $g(x) = f(x + 7)$

4. $y = \cos(x + 0.5)$

5. $g(x) = f(x - 2.3)$

6. $y = \sin\left(x + \frac{3\pi}{2}\right)$

Use the function $f(x)$ shown below. Graph each translation.



7. $g(x) = f(x + 2)$

8. $g(x) = f(x) + 2$

9. Writing Write a cosine function that has amplitude 2, period 2π, phase shift 3, and vertical shift −7.

Name_____ Class_____ Date_____

4-6 Practice (continued) *Form K*

Translating Sine and Cosine Functions

Graph each translation of $y = \sin x$ in the interval from 0 to 2π.

10. $y = \sin\left(x + \frac{\pi}{2}\right)$

11. $y = \sin x + 2$

Graph each function in the interval from 0 to 2π.

12. $y = \cos\left(x - \frac{\pi}{2}\right) + 2$

13. $y = \sin(x - \pi) - 1$

Write an equation for each of the following translations.

14. $y = \cos x$, 6 units down

15. $y = \sin x$, 2π units right

16. $y = \cos x$, $\frac{\pi}{2}$ units left

17. $y = \sin x$, 3 units up and 5 units right

18. $y = \cos x$, 2 units down and $\frac{\pi}{6}$ units left

19. Error Analysis Your classmate said that the function $y = \cos(x + 5)$ is a translation 5 units to the right. What error did he make? What translation does this function represent?

Name_____ Class_____ Date_____

4-7 Practice

Form K

Trigonometric Identities

Verify each identity. Give the domain of validity for each identity.

1. $\sin\theta\cot\theta = \cos\theta$

Write the equation in terms of sine and cosine: $\sin\theta$ (_____) $= \cos\theta$

Look at each part to determine where the identity is valid.

$\sin\theta$ and $\cos\theta$ are defined _____.

$\cot\theta$ is defined for all real numbers except _____.

The domain of validity is _____.

2. $\dfrac{\cos\theta}{\sec\theta} = \cos^2\theta$

3. $\sin\theta\cot\theta\sec\theta = 1$

4. $\dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta} = \sec\theta$

5. $\dfrac{\cot\theta}{\csc\theta} = \cos\theta$

6. $\dfrac{\sec\theta}{\tan\theta} = \csc\theta$

7. $\sin^2\theta\sec\theta + \cos\theta = \sec\theta$

8. $\cot^2\theta - \csc^2\theta = -1$

9. $\csc\theta - \sin\theta = \cos\theta\cot\theta$

10. $\dfrac{\csc\theta}{\cot\theta} = \sec\theta$

11. $\sin\theta(1 + \cot^2\theta) = \csc\theta$

12. $\dfrac{1 - \sin^2\theta}{\sin^2\theta} = \cot^2\theta$

13. $\dfrac{\cos\theta\csc\theta}{\tan\theta}\ \cot^2\theta$

Name_____ Class_____ Date_____

4-7 Practice (continued) Form K

Trigonometric Identities

Simplify each trigonometric expression.

14. $\dfrac{\sin\theta\csc\theta}{\cot\theta}$

To start, replace one expression with an expression containing sine, cosine, or tangent:

$\csc\theta = \dfrac{1}{\sin\theta}$

$\dfrac{\sin\theta\csc\theta}{\cot\theta} = \dfrac{\sin\theta \cdot \square}{\cot\theta} = \dfrac{\square}{\cot\theta}$

Then use a reciprocal identity: $\dfrac{\square}{\cot\theta} = \square$

$\dfrac{\sin\theta\csc\theta}{\cot\theta} = \square$

15. $\cos^2\theta\sec\theta\csc\theta$

16. $\cot\theta\tan\theta - \sec^2\theta$

17. $\cos\theta(1+\tan^2\theta)$

18. $\dfrac{\tan\theta}{\sec\theta}$

19. $\dfrac{\cot\theta\sin\theta}{\cos\theta}$

20. $\cos\theta + \sin\theta\tan\theta$

21. $\cos\theta\cot\theta\sin\theta$

22. $\dfrac{\csc\theta\cot\theta\cos\theta}{\cot^2\theta}$

23. Writing Explain the relationship between the Pythagorean Theorem and the Pythagorean identities.

24. Error Analysis A student writes on a quiz that the domain of validity for the expression $\dfrac{\sec\theta}{\sin\theta}$ is all real numbers except multiples of π. What is the student's error? What is the correct domain of validity?

Name_____ Class_____ Date_____

4-7 Examples

Reciprocal Trigonometric Functions

Match each of the following items with its reciprocal.

1. sine

2. cosine

3. tangent

A. secant

B. cotangent

C. cosecant

Find each value without using a calculator.

4. sec 2π

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\sec 2\pi = \frac{1}{\cos 2\pi} = \frac{\square}{\square} = \square$$

5. csc $\frac{5\pi}{6}$

6. cot $\frac{\pi}{3}$

7. csc $\frac{7\pi}{4}$

8. sec $\frac{4\pi}{3}$

9. cot $\frac{\pi}{4}$

Use a calculator to find the decimal values of the following expressions. Round your answers to the nearest thousandth. Remember to use the reciprocals of sine, cosine, and tangent.

10. sec 20

11. csc 3.4

12. cot (–4)

13. csc 62°

14. sec 286°

15. cot 165°

Name_____ Class_____ Date_____

4-7 Examples

Reciprocal Trigonometric Functions

Use the table of values to solve the following problem.

16. What are the graphs of $y = \cos x$ and $y = \sec x$ in the interval from 0 to 2π?

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$\cos x$	1	0.9	0.5	0	–0.5	–0.9	–1	–0.9	–0.5	0	–0.5	–0.9	1
$\sec x$	1	1.2	2	undef	–2	–1.2	–1	–1.2	–2	undef	2	1.2	1

Use the graph of secant, cosecant, or cotangent to find each value. Round your answers to the nearest thousandth.

17. sec 40°

18. cot 50°

19. csc 75°

20. cot 120°

21. csc 160°

22. cot 80°

Use a graphing calculator to solve the following problem.

23. A spotlight sits on top of a building and shines on a bush. The beam of light shines in a path that makes an angle of 50° with the building. The distance in feet from the spotlight to the bush is modeled by the function $d = 125 \sec \theta$. What is the distance from the bush to the spotlight rounded to the nearest foot?