Name_____ Class_____ Date_____

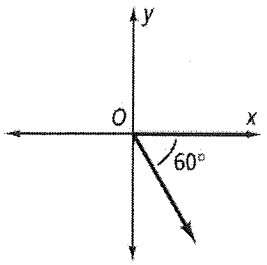
4-1 Practice

Form G

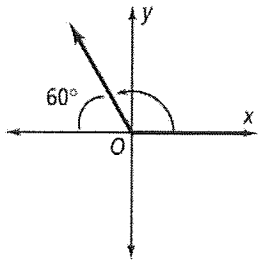
Angles and the Unit Circle

Find the measure of each angle in standard position.

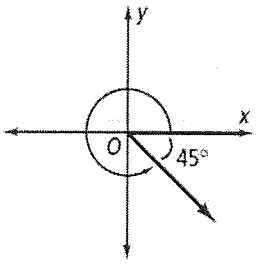
000 **1.**



2.



3.



Sketch each angle in standard position.

4. 100° **5.** 210° **6.** −45° **7.** −90°

8. −330° **9.** −180° **10.** −145° **11.** 60°

Find the measure of an angle between 0° and 360° coterminal with each given angle.

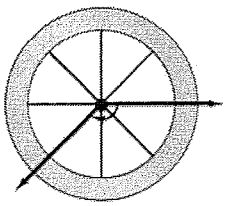
12. −100°	**13.** −60°	**14.** −225°	**15.** −145°	**16.** 372°
17. −15°	**18.** 482°	**19.** 484°	**20.** −20°	**21.** 421°
22. 409°	**23.** −38°	**24.** 376°	**25.** −210°	**26.** 387°
27. 390°	**28.** 660°	**29.** 440°	**30.** −170°	**31.** 370°
32. −700°	**33.** 458°	**34.** 480°	**35.** 406°	**36.** −120°
37. 460°	**38.** −222°	**39.** −330°	**40.** −127°	**41.** 377°

Name_____ Class_____ Date _____

4-1 Practice (continued) *Form G*

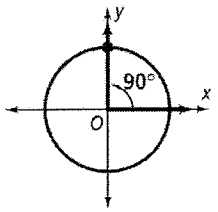
Angles and the Unit Circle

42. The spokes shown on the bicycle wheel at the right form an angle. Estimate the measures of two coterminal angles that coincide with the angle at the right.

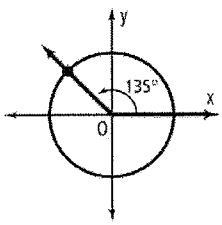


Find the exact values of the cosine and sine of each angle. Then find the decimal values. Round your answers to the nearest hundredth.

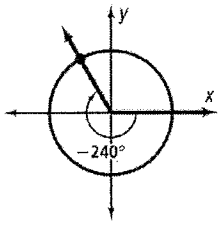
43.



44.



45.



46. 45° **47.** −150° **48.** 720°

Graphing Calculator For each angle θ, find the values of cos θ and sin θ. Round your answers to the nearest hundredth.

49. 225° **50.** −225° **51.** −45°

52. 330° **53.** −330° **54.** 150°

Open-Ended Find a positive and a negative coterminal angle for the given angle.

55. 50° **56.** −130° **57.** −680°

58. 395° **59.** −38° **60.** −434°

61. a. Suppose you know the terminal side of angle θ lies in Quadrant II. What is the sign of cos θ? sin u?

b. Writing Describe the reasoning you followed to answer part (a).

Name_____ Class_____ Date_____

4-2 Practice

Form G

Radian Measure

Write each measure in radians. Express your answer in terms of π and as a decimal rounded to the nearest hundredth.

1. 45° **2.** 90° **3.** 30° **4.** −150°

5. 180° **6.** −240° **7.** 270° **8.** 300°

Write each measure in degrees. Round your answer to the nearest degree, if necessary.

9. $\frac{\pi}{6}$ radians **10.** $-\frac{7\pi}{6}$ radians **11.** $\frac{7\pi}{4}$ radians

12. −4 radians **13.** 1.8 radians **14.** 0.45 radians

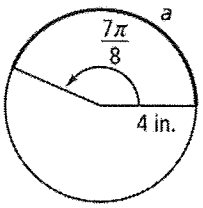
The measure θ of an angle in standard position is given. Find the exact values of $\cos\theta$ and $\sin\theta$ for each angle measure.

15. $\frac{\pi}{6}$ **16.** $\frac{\pi}{3}$ **17.** $-\frac{3\pi}{4}$

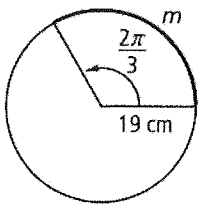
18. $\frac{7\pi}{4}$ **19.** $\frac{11\pi}{6}$ **20.** $-\frac{2\pi}{3}$

Use each circle to find the length of the indicated arc. Round your answer to the nearest tenth.

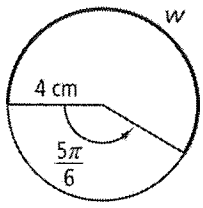
21.



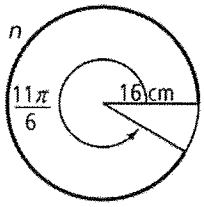
22.



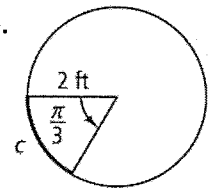
23.



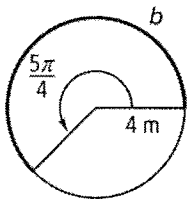
24.



25.



26.



4-2 Practice (continued) Form G

Radian Measure

27. The minute hand of a clock is 8 in. long.

a. What distance does the tip of the minute hand travel in 10 min?

b. What distance does the tip of the minute hand travel in 40.5 min?

c. What distance does the tip of the minute hand travel in 3.25 h?

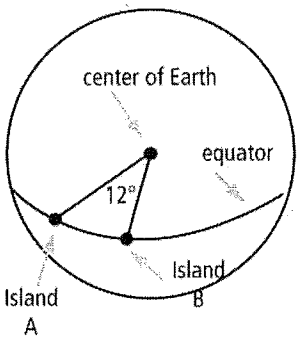
d. Reasoning After approximately how many hours has the tip of the minute hand traveled 100 ft?

28. A 0.8 m pendulum swings through an angle of 86°. What distance does the tip of the pendulum travel?

29. A scientist studies two islands shown at the right. The distance from the center of the Earth to the equator is about 3960 mi.

a. What is the measure in radians of the central angle that intercepts the arc along the equator between the islands?

b. About how far apart are the two islands?



Determine the quadrant or axis where the terminal side of each angle lies.

30. $\frac{\pi}{5}$ **31.** $-\frac{5\pi}{2}$ **32.** $\frac{5\pi}{3}$ **33.** $\frac{8\pi}{7}$

Draw an angle in standard position with each given measure. Then find the values of the cosine and sine of the angle to the nearest hundredth.

34. $\frac{5\pi}{4}$ **35.** -3π **36.** $\frac{2\pi}{9}$

37. Error Analysis A student wanted to convert 75° to radians. His calculation is shown at the right. What error did he make? What is the correct conversion?

$$\frac{(75\times180)}{\pi} \approx 4297.18 \text{ radians}$$

Name_____ Class _____ Date_____

Practice

Unit 4

Lessons 4-3 through 4-5

Identify the amplitude or asymptotes, and the period for each function.

1. $y = 4 \sin 3x$

2. $y = \cos 4x$

3. $y = \frac{1}{3} \tan \pi x$

4. $y = 2 \cos \frac{x}{4}$

5. $y = 3 \tan x$

6. $y = \frac{1}{9} \sin 5x$

Sketch the graph of one cycle of each function.

7. $y = 2 \cos x$

8. $y = 3 \sin 2x$

9. $y = \tan \frac{x}{2}$

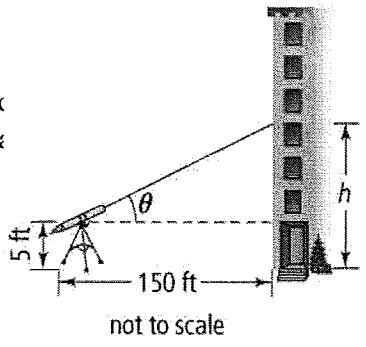
10. $y = -\sin 3x$

11. $y = \cos 4x$

12. $y = -2 \tan \pi x$

13. Earthquakes under the ocean can sometimes cause a dangerous wave called a tsunami. You can model the motion of a tsunami with the function $f(x) = a \cos bx$. Write an equation that models a tsunami that travels at 120 ft/s, has a period of 20 s, and has an amplitude of 60 ft.

14. Owen is using a telescope to measure a tall building down the telescope, Owen can see the point on the building that ground. The relationship between the position of the telesc can be modeled by $h = 150 \tan \theta + 5$. Owen changed the a 75°. How much higher is the point on the building that he



not to scale

Name_____ Class_____ Date_____

4-6 Practice

Form G

Translating Sine and Cosine Functions

Determine the value of h in each translation. Describe each phase shift (use a phrase like *3 units to the left*).

1. $g(x) = f(x + 2)$

2. $g(x) = f(x - 1)$

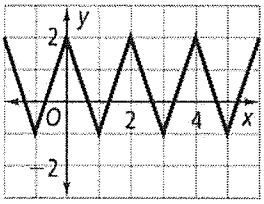
3. $h(t) = f(t + 1.5)$

4. $f(x) = g(x - 1)$

5. $y = \cos\left(x - \frac{\pi}{2}\right)$

6. $y = \cos(x + \pi)$

Use the function $f(x)$ at the right. Graph each translation.



7. $f(x) - 5$

8. $f(x + 3)$

Graph each translation of $y = \cos x$ in the interval from 0 to 2π.

9. $y = \cos(x + 4)$

10. $y = \cos x + 3$

11. $y = \cos\left(x + \frac{\pi}{6}\right)$

Describe any phase shift and vertical shift in the graph.

12. $y = 3\cos x + 2$

13. $y = 2\cos(x - 1) + 3$

14. $y = \sin\left(x + \frac{3\pi}{2}\right) - 1$

Graph each function in the interval from 0 to 2π.

15. $y = 3\sin\left(x - \frac{\pi}{4}\right) + 2$

16. $y = \cos\left(x + \frac{\pi}{2}\right) - 1$

17. $y = \sin(x - \pi) + 2$

18. $y = \cos\frac{1}{2}x + 1$

19. $y = \sin 2\left(x - \frac{\pi}{3}\right)$

20. $y = -\cos 2\left(x + \frac{\pi}{4}\right)$

Name_____ Class_____ Date_____

4-6 Practice (continued) *Form G*

Translating Sine and Cosine Functions

Write an equation for each translation.

21. $y = \sin x$, 2 units down

22. $y = \cos x$, π units to the left

23. $y = \cos x$, $\frac{\pi}{4}$ units up

24. $y = \sin x$, 3.2 units to the right

25. $y = \sin x$; 3 units to the left, 1 unit down

26. $y = \cos x$; $\frac{\pi}{2}$ units to the right, 2 units up

27. The table below shows the temperatures at a weather station on several days of the year.

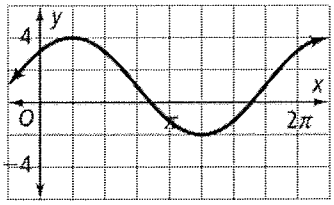
Day of the Year	15	48	73	104	136	169	196	228	257	290	323	352
Temp. (°F)	76	73	75	79	82	87	90	89	88	87	83	79

a. Plot the data

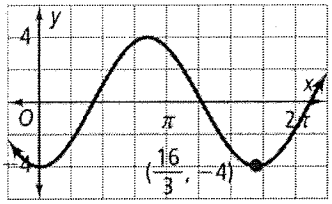
b. Write a cosine model for the data

Write a cosine function for each graph. Then write a sine function for each graph.

28.



29.



30. a. Write a sine function to model the weather station data in Exercise 27.

b. Writing How do the cosine and sine models differ?

c. Estimation Use your sine model to estimate the temperature at the weather station on December 31 (day 365).

Name_____ Class_____ Date_____

4-7 Practice

Form G

Trigonometric Identities

Verify each identity. Give the domain of validity for each identity.

1. $\sin\theta \sec\theta \cot\theta = 1$

2. $\csc\theta = \cot\theta \sec\theta$

3. $\dfrac{\sin\theta}{\csc\theta} = \sin^2\theta$

4. $\cos\theta \csc\theta \tan\theta = 1$

5. $\sin\theta \tan\theta + \cos\theta = \sec\theta$

6. $\dfrac{\csc\theta}{\cot\theta} = \sec\theta$

7. $\sec\theta = \tan\theta \csc\theta$

8. $\tan\theta + \cot\theta = \sec\theta \csc\theta$

9. $\tan^2\theta + 1 = \sec^2\theta$

10. $\cos\theta \cot\theta + \sin\theta = \csc\theta$

11. $\dfrac{\sec\theta}{\csc\theta} = \tan\theta$

12. $\sec\theta \cot\theta = \csc\theta$

13. $\sec^2\theta - \tan^2\theta = 1$

14. $\sec\theta = \csc\theta \tan\theta$

15. $\dfrac{\sin\theta + \cos\theta}{\sin\theta} = 1 + \cot\theta$

16. $\cos\theta(\sec\theta - \cos\theta) = \sin^2\theta$

17. $\cot\theta \sec\theta = \csc\theta$

18. $(1 - \sin\theta)(1 + \sin\theta) = \cos^2\theta$

Simplify each trigonometric expression.

19. $1 - \sec^2\theta$

20. $\dfrac{\sec\theta}{\tan\theta}$

Name_____ Class_____ Date_____

4-7 Practice (continued) Form G

Trigonometric Identities

Simplify each trigonometric expression.

21. $\csc \theta \tan \theta$

22. $\sec \theta \cos^2 \theta$

23. $\csc^2 \theta - \cot^2 \theta$

24. $1 - \sin^2 \theta$

25. $\tan \theta \cot \theta$

26. $\cos \theta \cot \theta + \sin \theta$

27. $\cos \theta \tan \theta$

28. $\dfrac{\sin \theta \cot \theta}{\cos \theta}$

29. $\sec \theta \tan \theta \csc \theta$

30. $\sec \theta \cot \theta$

31. $\dfrac{\sin \theta}{\csc \theta} + \dfrac{\cos \theta}{\sec \theta}$

32. $\dfrac{\tan \theta \csc \theta}{\sec \theta}$

33. $\cot^2 \theta - \csc^2 \theta$

34. $\dfrac{\cot \theta}{\csc \theta}$

Express the first trigonometric function in terms of the second.

35. $\csc \theta$, $\sin \theta$

36. $\cot \theta$, $\tan \theta$

37. $\sec \theta$, $\cos \theta$

38. $\cos \theta$, $\sin \theta$

39. Writing Which side of the equation below should you transform to verify the identity? Explain. $\dfrac{\cos^2 \theta + \tan^2 \theta - 1}{\sin^2 \theta} = \tan^2 \theta$

Name_____ Class_____ Date_____

4-7 Examples

Reciprocal Trigonometric Functions

Match each of the following items with its reciprocal.

1. sine

2. cosine

3. tangent

A. secant

B. cotangent

C. cosecant

Find each value without using a calculator.

4. $\sec 2\pi$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\sec 2\pi = \frac{1}{\cos 2\pi} = \frac{\square}{\square} = \square$$

5. $\csc \frac{5\pi}{6}$

6. $\cot \frac{\pi}{3}$

7. $\csc \frac{7\pi}{4}$

8. $\sec \frac{4\pi}{3}$

9. $\cot \frac{\pi}{4}$

Use a calculator to find the decimal values of the following expressions. Round your answers to the nearest thousandth. Remember to use the reciprocals of sine, cosine, and tangent.

10. sec 20

11. csc 3.4

12. cot (–4)

13. csc 62°

14. sec 286°

15. cot 165°

Name_____ Class_____ Date_____

4-7 Examples

Reciprocal Trigonometric Functions

Use the table of values to solve the following problem.

16. What are the graphs of $y = \cos x$ and $y = \sec x$ in the interval from 0 to 2π?

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$\cos x$	1	0.9	0.5	0	–0.5	–0.9	–1	–0.9	–0.5	0	–0.5	–0.9	1
$\sec x$	1	1.2	2	undef	–2	–1.2	–1	–1.2	–2	undef	2	1.2	1

Use the graph of secant, cosecant, or cotangent to find each value. Round your answers to the nearest thousandth.

17. sec 40°

18. cot 50°

19. csc 75°

20. cot 120°

21. csc 160°

22. cot 80°

Use a graphing calculator to solve the following problem.

23. A spotlight sits on top of a building and shines on a bush. The beam of light shines in a path that makes an angle of 50° with the building. The distance in feet from the spotlight to the bush is modeled by the function $d = 125 \sec \theta$. What is the distance from the bush to the spotlight rounded to the nearest foot?

Name_____ Class _____ Date_____

5-7 Reteaching (continued)

Trigonometric Identities

To verify an identity, you can transform one side of the equation until it is the same as the other side. Begin by writing all of the functions in terms of sine and cosine.

Once you choose a side of the equation to transform, do not work with the other side of the equation. Raising both sides of the equation to a power or dividing both sides of the equation by a trigonometric expression can introduce extraneous solutions.

Problem

Verify the identity $1 + \cot^2 \theta = \csc^2 \theta$.

$1 + \cot^2 \theta = 1 + \left(\frac{\cos \theta}{\sin \theta}\right)^2$ Cotangent Identity

$= 1 + \frac{\cos^2 \theta}{\sin^2 \theta}$ Simplify.

$= \frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta}$ Write the fractions with common denominators.

$= \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta}$ Add.

$= \frac{1}{\sin^2 \theta}$ Pythagorean Identity

$= \csc^2 \theta$ Reciprocal Identity

Exercises

Verify each identity.

7. $\cot \theta \tan \theta = 1$

8. $\cos \theta \sec \theta = 1$

9. $\csc \theta \sin \theta + \cot^2 \theta = \csc^2 \theta$

10. $\sin \theta (1 + \cot^2 \theta) = \csc \theta$

11. $\sec \theta \cot \theta = \csc \theta$

12. $\sec^2 \theta - \sec^2 \theta \cos^2 \theta = \tan^2 \theta$

13. $\cot \theta \tan \theta + \tan^2 \theta = \sec^2 \theta$

14. $\csc^2 \theta - \cot^2 \theta = 1$

15. $\sin \theta + \cos \theta \cot \theta = \csc \theta$

16. $\frac{\sec \theta - \cos \theta}{\sec \theta} = \sin^2 \theta$

17. $\cot \theta \sec \theta \sin \theta = 1$

18. $\tan \theta (\sin \theta - \csc \theta) = -\cos \theta$

Name_____ Class _____ Date_____

5-7 Reteaching

Trigonometric Identities

A *trigonometric identity* is a trigonometric equation that is true for all values of the variable except those that cause the expressions on either side of the equal sign to be undefined. You can use the trigonometric identities below to replace complicated-looking expressions with much simpler ones.

Reciprocal Identities	$\csc\theta = \frac{1}{\sin\theta}$	$\cot\theta = \frac{1}{\tan\theta}$	$\sec\theta = \frac{1}{\cos\theta}$
Pythagorean Identities	$\sin^2\theta + \cos^2\theta = 1$	$\tan^2\theta + 1 = \sec^2\theta$	$1 + \cot^2\theta = \csc^2\theta$

Problem

What is $\cot\theta(\tan\theta + \cot\theta)$ expressed in simplified terms?

Use the identities to rewrite $\cot\theta$ and $\tan\theta$.

$$\cot\theta\left(\tan\theta + \cot\theta\right) = \frac{1}{\tan\theta}\left(\tan\theta + \frac{1}{\tan\theta}\right)$$ Write $\cot\theta$ in terms of $\tan\theta$.

$$= \frac{1}{\tan\theta}\left(\tan\theta\right) + \left(\frac{1}{\tan\theta}\right)^2$$ Distribute.

$$= 1 + \left(\frac{1}{\tan\theta}\right)^2$$ Simplify.

$$= 1 + \cot^2\theta$$ Reciprocal Identity

$$= \csc^2\theta$$ Pythagorean Identity

Exercises

Simplify each expression.

1. $\cot\theta \sin\theta$

2. $\tan\theta \cos\theta$

3. $\csc\theta \sin\theta$

4. $\cos\theta \sin\theta \sec\theta$

5. $\sin\theta + \cot\theta \cos\theta$

6. $\csc^2\theta - \cot^2\theta$

Name_____ Class _____ Date_____

5-7 Reteaching

Trigonometric Identities

A *trigonometric identity* is a trigonometric equation that is true for all values of the variable except those that cause the expressions on either side of the equal sign to be undefined. You can use the trigonometric identities below to replace complicated-looking expressions with much simpler ones.

Reciprocal Identities	$\csc\theta = \frac{1}{\sin\theta}$	$\cot\theta = \frac{1}{\tan\theta}$	$\sec\theta = \frac{1}{\cos\theta}$
Pythagorean Identities	$\sin^2\theta + \cos^2\theta = 1$	$\tan^2\theta + 1 = \sec^2\theta$	$1 + \cot^2\theta = \csc^2\theta$

Problem

What is $\cot\theta(\tan\theta + \cot\theta)$ expressed in simplified terms?

Use the identities to rewrite $\cot\theta$ and $\tan\theta$.

$$\cot\theta(\tan\theta + \cot\theta) = \frac{1}{\tan\theta}\left(\tan\theta + \frac{1}{\tan\theta}\right)$$ Write $\cot\theta$ in terms of $\tan\theta$.

$$= \frac{1}{\tan\theta}(\tan\theta) + \left(\frac{1}{\tan\theta}\right)^2$$ Distribute.

$$= 1 + \left(\frac{1}{\tan\theta}\right)^2$$ Simplify.

$$= 1 + \cot^2\theta$$ Reciprocal Identity

$$= \csc^2\theta$$ Pythagorean Identity

Exercises

Simplify each expression.

1. $\cot\theta \sin\theta$

2. $\tan\theta \cos\theta$

3. $\csc\theta \sin\theta$

4. $\cos\theta \sin\theta \sec\theta$

5. $\sin\theta + \cot\theta \cos\theta$

6. $\csc^2\theta - \cot^2\theta$

Name_____ Class _____ Date_____

5-7 Reteaching (continued)

Trigonometric Identities

To verify an identity, you can transform one side of the equation until it is the same as the other side. Begin by writing all of the functions in terms of sine and cosine.

Once you choose a side of the equation to transform, do not work with the other side of the equation. Raising both sides of the equation to a power or dividing both sides of the equation by a trigonometric expression can introduce extraneous solutions.

Problem

Verify the identity $1 + \cot^2 \theta = \csc^2 \theta$.

$1 + \cot^2 \theta = 1 + \left(\frac{\cos \theta}{\sin \theta}\right)^2$ Cotangent Identity

$= 1 + \frac{\cos^2 \theta}{\sin^2 \theta}$ Simplify.

$= \frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta}$ Write the fractions with common denominators.

$= \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta}$ Add.

$= \frac{1}{\sin^2 \theta}$ Pythagorean Identity

$= \csc^2 \theta$ Reciprocal Identity

Exercises

Verify each identity.

7. $\cot \theta \tan \theta = 1$

8. $\cos \theta \sec \theta = 1$

9. $\csc \theta \sin \theta + \cot^2 \theta = \csc^2 \theta$

10. $\sin \theta (1 + \cot^2 \theta) = \csc \theta$

11. $\sec \theta \cot \theta = \csc \theta$

12. $\sec^2 \theta - \sec^2 \theta \cos^2 \theta = \tan^2 \theta$

13. $\cot \theta \tan \theta + \tan^2 \theta = \sec^2 \theta$

14. $\csc^2 \theta - \cot^2 \theta = 1$

15. $\sin \theta + \cos \theta \cot \theta = \csc \theta$

16. $\frac{\sec \theta - \cos \theta}{\sec \theta} = \sin^2 \theta$

17. $\cot \theta \sec \theta \sin \theta = 1$

18. $\tan \theta (\sin \theta - \csc \theta) = -\cos \theta$